Angles

As derived from the Greek language, the word **trigonometry** means "measurement of triangles." Initially, trigonometry dealt with relationships among the sides and angles of triangles and was used in the development of astronomy, navigation, and surveying.

With the development of calculus and the physical sciences in the 17th century, a different perspective arose—one that viewed the classic trigonometric relationships as *functions* with the set of real numbers as their domains.

Angles

Consequently, the applications of trigonometry expanded to include a vast number of physical phenomena involving rotations and vibrations, including the following.

- sound waves
- light rays
- planetary orbits
- vibrating strings
- pendulums
- orbits of atomic particles

The math is not difficult, it is all about the

VOCABULARY

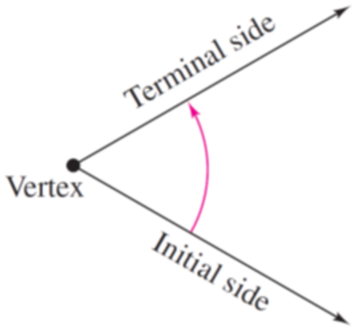
Angles

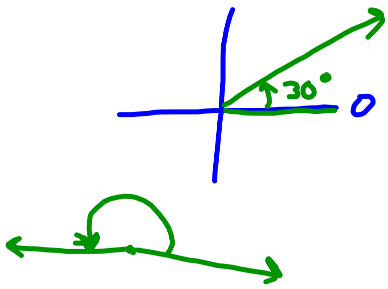
The approach in this text incorporates *both* perspectives, starting with angles and their measure.

An **angle** is determined by rotating a ray (half-line) about its endpoint. The starting position of the ray is the **initial side** of the angle, and the position after rotation is the **terminal side,** as shown in Figure 4.1.

30°

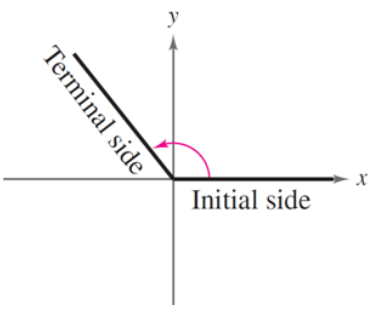
180°





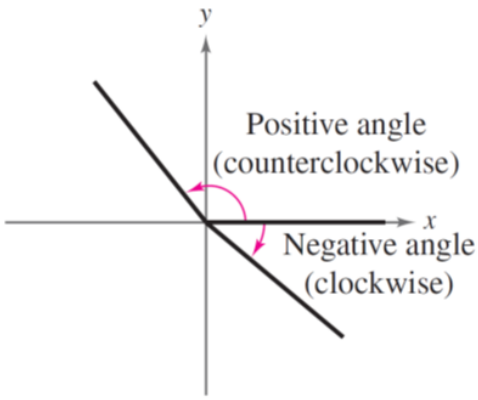
Angles

The endpoint of the ray is the **vertex** of the angle. This perception of an angle fits a coordinate system in which the origin is the vertex and the initial side coincides with the positive *x*-axis. Such an angle is in **standard position,** as shown in Figure 4.2.



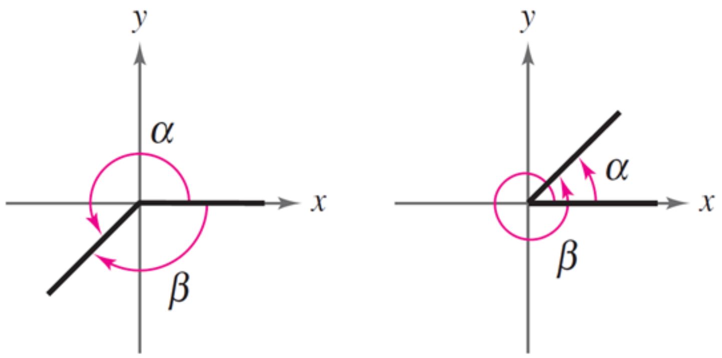
Angles

Positive angles are generated by counterclockwise rotation, and **negative angles** by clockwise rotation, as shown in Figure 4.3.



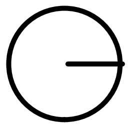
Angles

Angles are labeled with Greek letters such as α (alpha), β (beta), and (theta), as well as uppercase letters such as A,B, and C. In Figure 4.4, note that angles α and β have the same initial and terminal sides. Such angles are **coterminal.**



What do you think is the definition of coterminal angles?

Draw a circle with the radius of your given piece of string. Open the compass the length of the string to create your circle. The length of the string represents your radius.

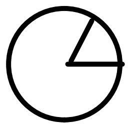


Connect the center of the circle to the circle.

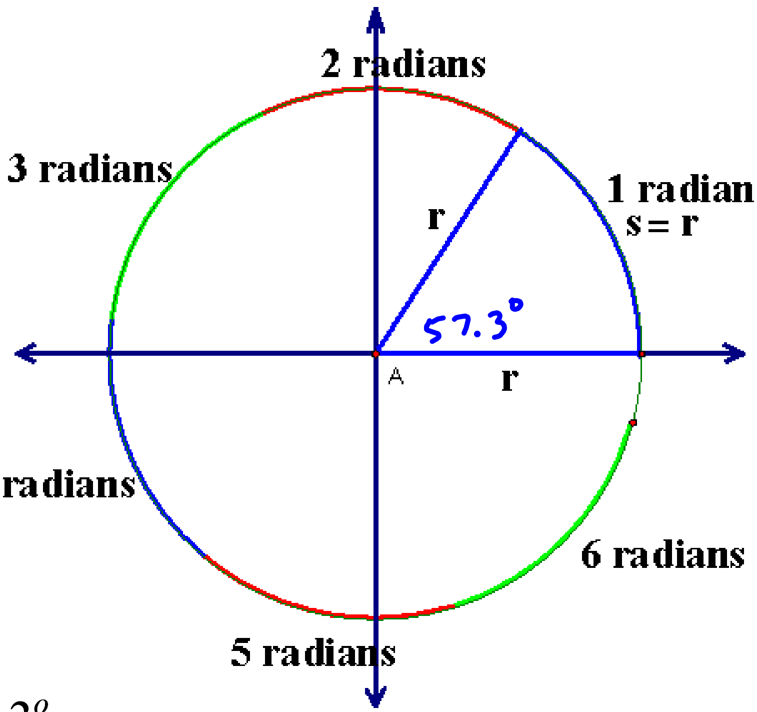
Now measure how many times your string will go around the circumference of the circle. Make a mark and number for each section.

How many strings does it take for the entire circle?

What do you notice?



Use a protractor to measure the degree of your angle created by your string.

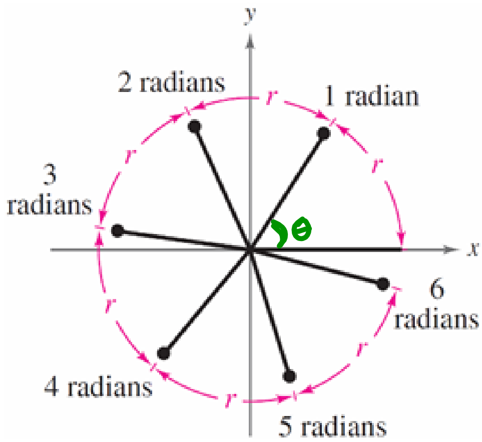


$$\frac{360^o}{2\pi} = 57.3^o$$

Radian Measure

Moreover, because

$$2\pi \approx 6.28$$

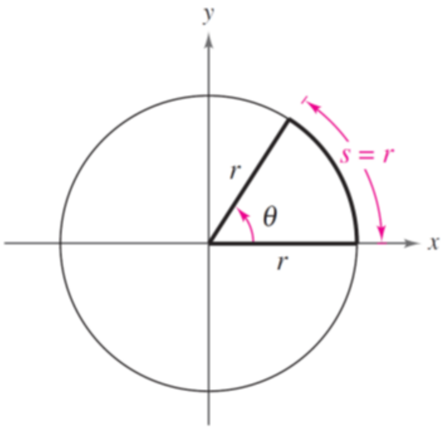


One radian is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle.

Radian Measure

The **measure of an angle** is determined by the amount of rotation from the initial side to the terminal side. One way to measure angles is in *radians*.

This type of measure is especially useful in calculus. To define a radian, you can use a **central angle** of a circle, one whose vertex is the center of the circle, as shown in Figure 4.5.



Arc length = radius when θ = 1 radian.

Radian Measure

Definition of Radian

One **radian** (rad) is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. (See Figure 4.5.) Algebraically this means that

$$\theta = \frac{s}{r}$$

where θ is measured in radians.

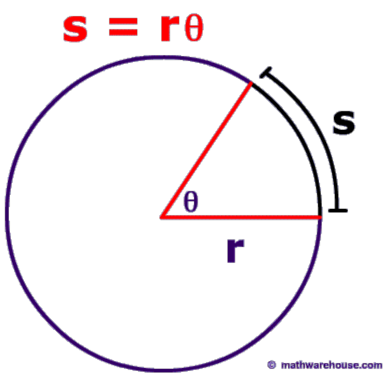
Because the circumference of a circle is $2\pi r$ units, it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of

$$s = 2\pi r.$$

$s = r\,\theta$

Arc Length = radius

Must be in **RADIANS**

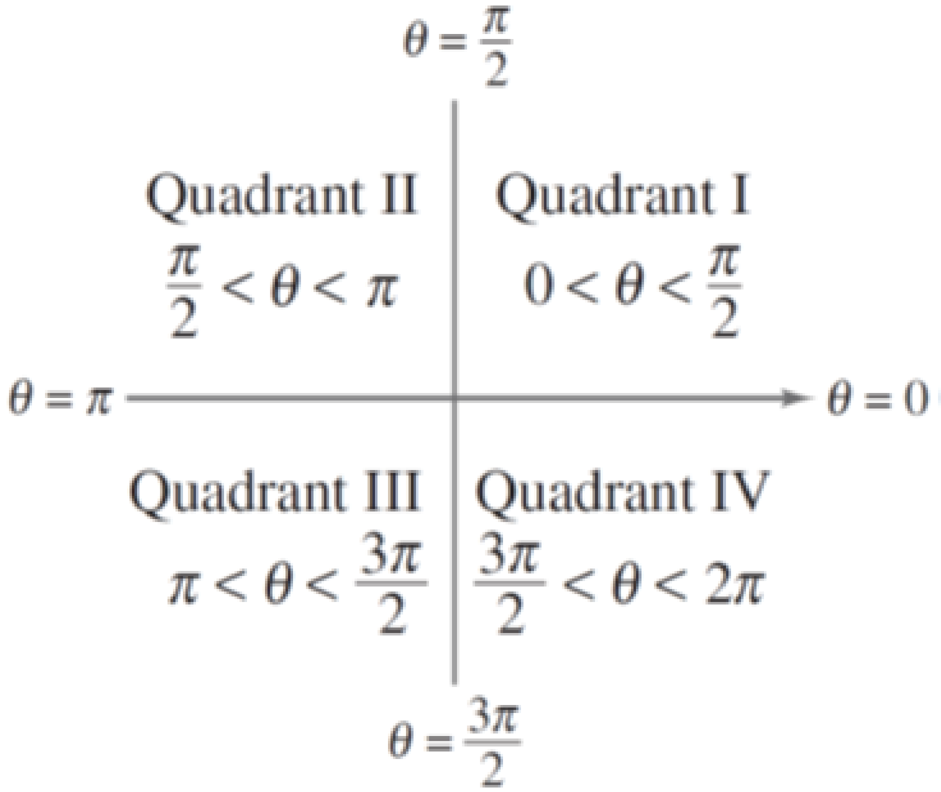


Radian Measure

Two angles are coterminal when they have the same initial and terminal sides. For instance, the angles 0 and 2π are coterminal, as are the angles $\pi/6$ and $13\pi/6$.

A given angle θ has infinitely many coterminal angles. For instance $\theta = \pi/6$, is coterminal with

$\frac{\pi}{6} + 2n\pi$, where is *n* an integer.

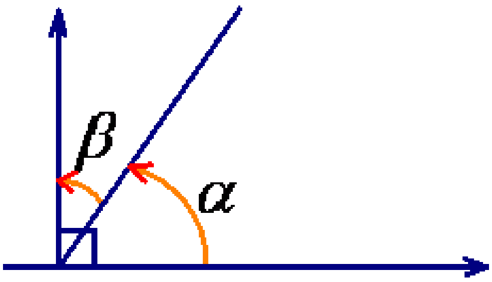

$\theta = \frac{\pi}{2}$
Quadrant II
$\frac{\pi}{2} < \theta < \pi$
Quadrant I
$0 < \theta < \frac{\pi}{2}$
$\theta = \pi$
$\theta = 0$
Quadrant III
$\pi < \theta < \frac{3\pi}{2}$
Quadrant IV
$\frac{3\pi}{2} < \theta < 2\pi$
$\theta = \frac{3\pi}{2}$

Degree Measure

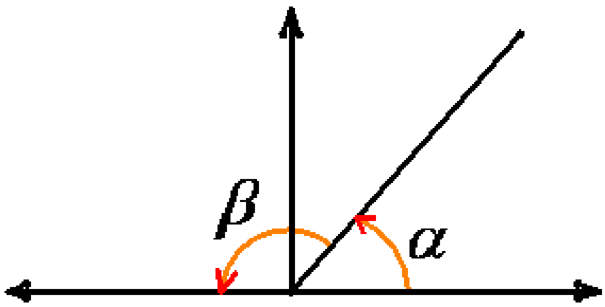
Because 2π radians corresponds to one complete revolution, degrees and radians are related by the equations

$360° = 2\pi$ rad and $180° = \pi$ rad.

	Degrees	Radians
Full revolution	360^{o}	2π
Half revolution	180^{o}	π
Acute angles	0^{o} and 90^{o}	0 and $\frac{\pi}{2}$
Obtuse angles	90^{o} and 180^{o}	$\frac{\pi}{2}$ and π
Complementary angles	add to 90^{o}	add to $\frac{\pi}{2}$
Supplementary angles	add to 180^{o}	add to π



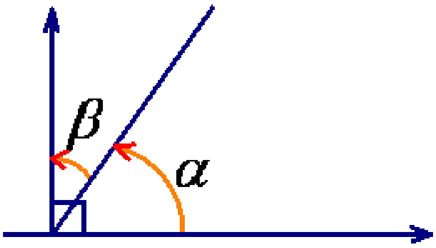
Complementary angles



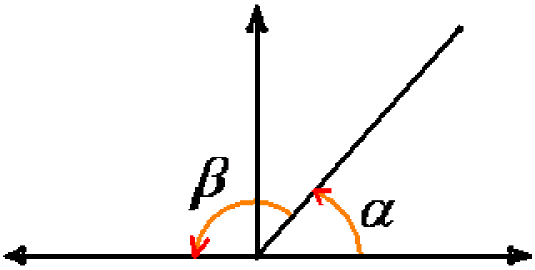
Supplementary angles

Two positive angles and are **complementary** **if their sum is____________.**

Two positive angles are **supplementary** **if their sum is ________________.**



Complementary angles



Supplementary angles

Find complement to $\frac{\pi}{3}$

$\frac{\pi}{2} - \frac{\pi}{3}$

Find supplement to $\frac{\pi}{3}$

Find complement to $\frac{2\pi}{3}$

Find supplement to $\frac{2\pi}{3}$

DMS- Degree, Minute and Second

To convert a decimal to degree minute second

2nd Angle #4

Example:
43.56 convert to DMS

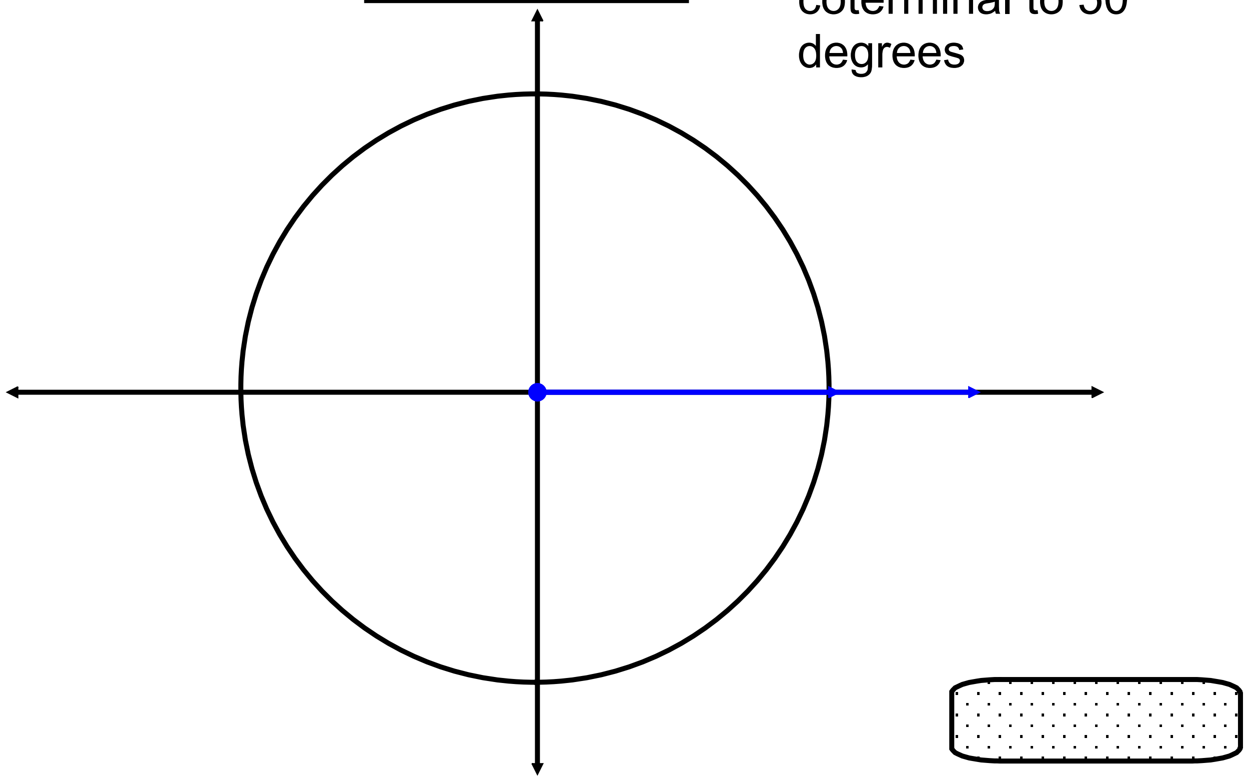
To type Degree Minute Second into calculator.

$43^{o}33'36''$

2nd Angle #1
2nd Angle #2
Alpha +

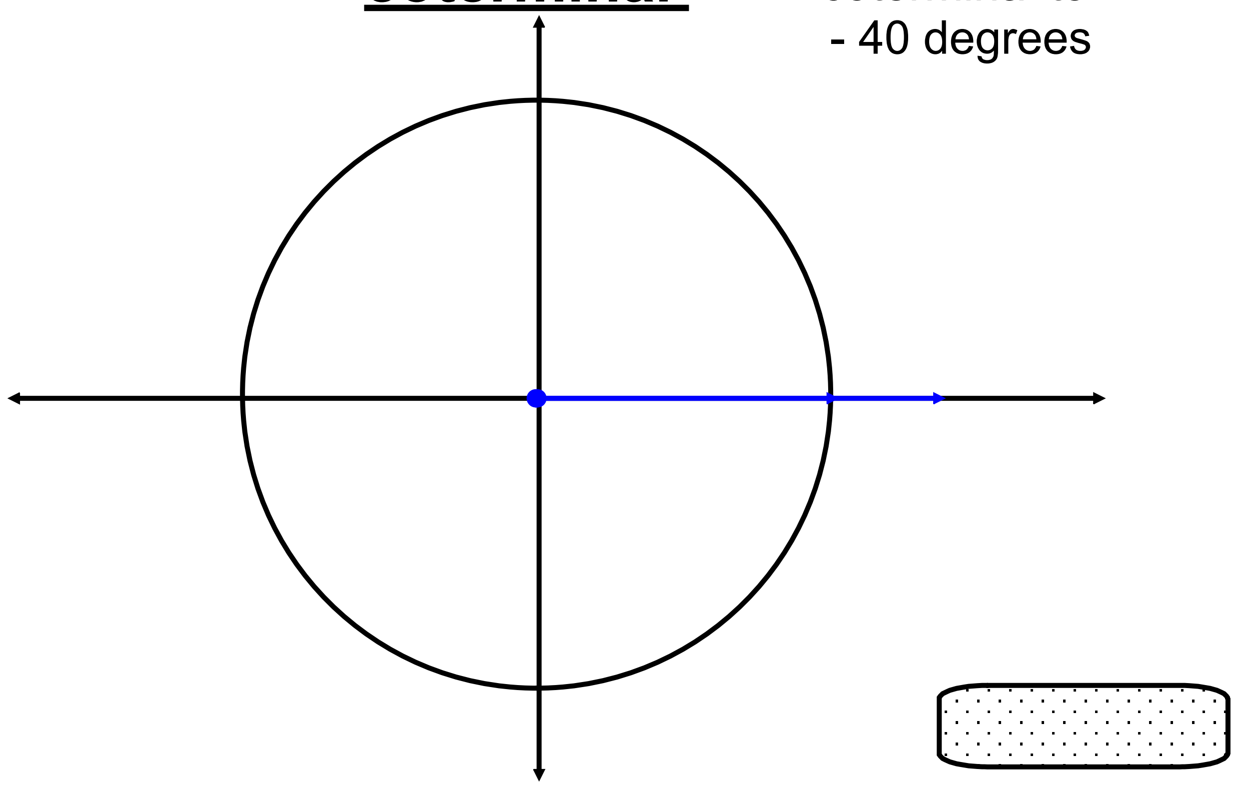
<u>coterminal</u>

coterminal to 50 degrees



<u>coterminal</u>

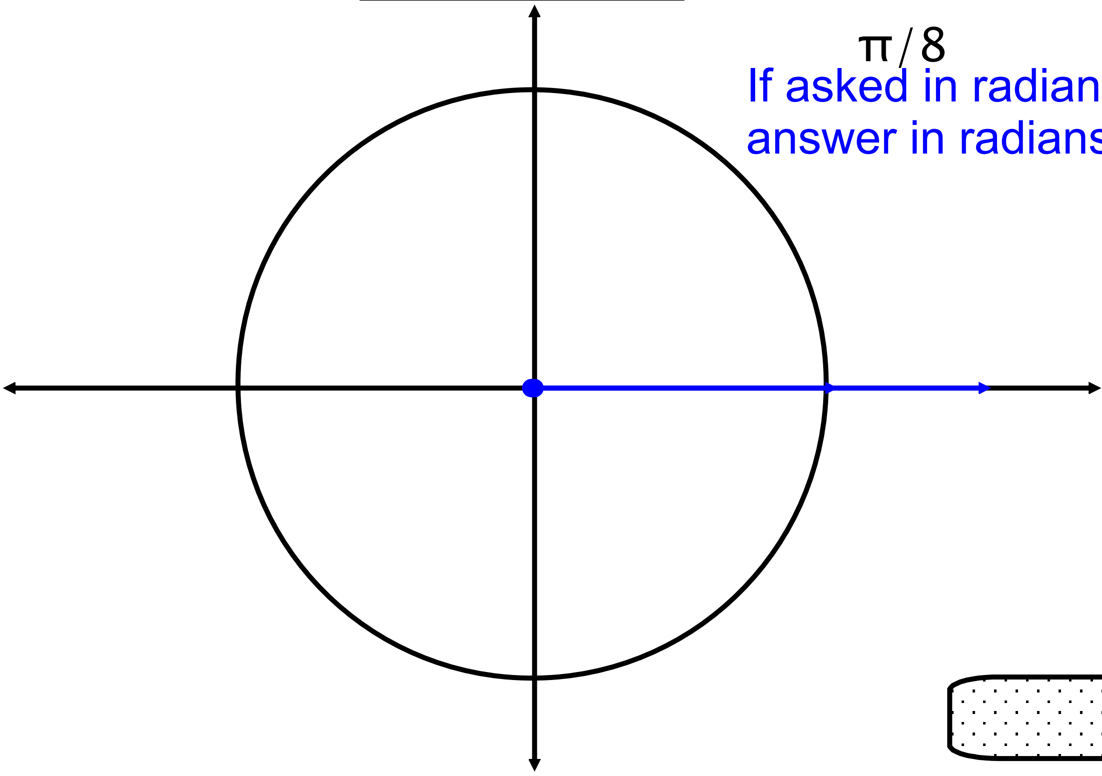
coterminal to
- 40 degrees



coterminal

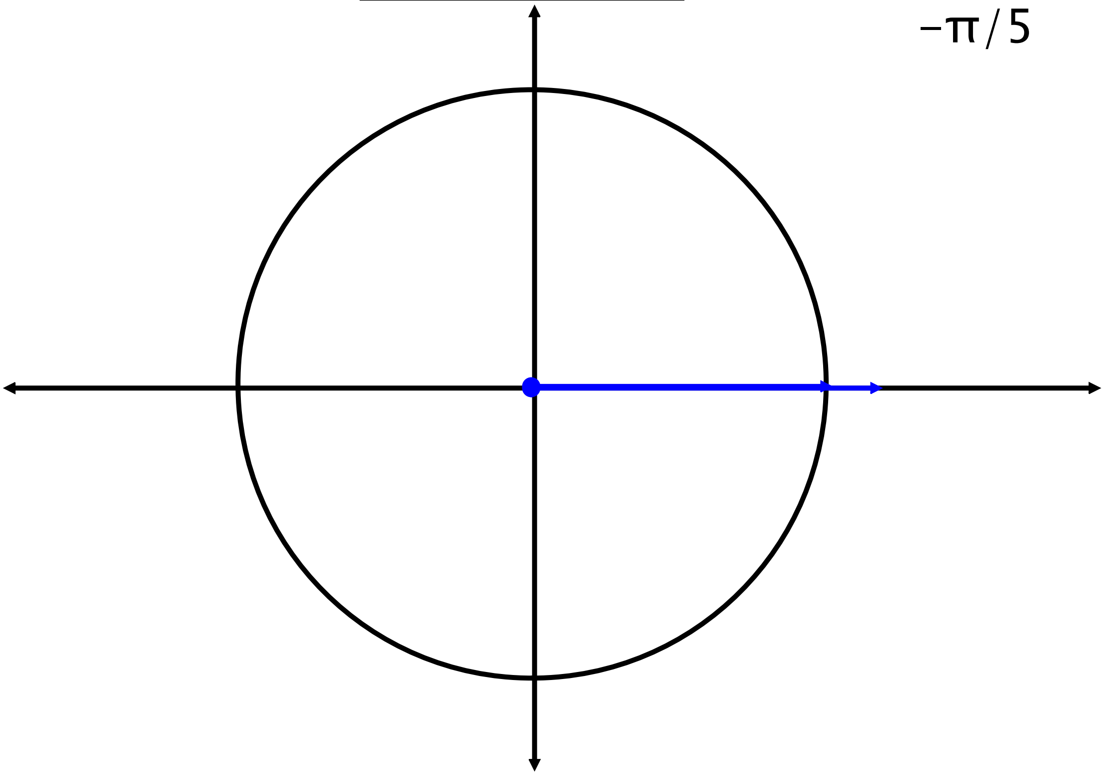
coterminal to $\pi/8$

If asked in radians, give answer in radians!



<u>coterminal</u>

coterminal to
$-\pi/5$



Conversions Between Degrees and Radians

1. To convert degrees to radians, multiply degrees by $\dfrac{\pi\ rad}{180^\circ}$

2. To convert radians to degrees, multiply radians by $\dfrac{180^\circ}{\pi\ rad}$

To apply these two conversion rules, use the basic relationship π radians = 180°.

Convert degrees to radians
or radians to degrees

MULTIPLY by ONE

$$\frac{180^\circ}{\pi} = 1 = \frac{\pi}{180^\circ}$$

angle

positive angle

terminal side

vertex

initial side

negative angle

coterminal

standard postition

determined by rotating a ray about its endpoint

is the starting position of the rotated ray in the formation of an angle.

is the position of the ray after the rotation when an angle is formed.

is the endpoint of the ray used in the formation an angle

the angle's vertex is at the origin of a coordinate system and its initial side coincides with the positive x-axis

counterclockwise rotation

clockwise rotation

two angles that have the same initial side and the same terminal side

You can convert any degree or radian, it doesn't have to be one of the memorized points.

Convert $9\pi/8$ radians to degrees
Convert 2 radians (doesn't always have to have π) to degrees

does 2 radians have a complement?

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The circular blade on a saw has a diameter of 7.5 in and rotates at 2400 revolutions per minute.

a. Find the angular speed in radians per second.

b. Find the linear speed of the saw teeth (in ft/sec) as they contact the wood being cut.

a. $$\frac{Revolutions}{second} = \quad = $$

Angular speed =

b. Radius of saw blade =

Radius in feet

$$speed = \frac{s}{t} = \frac{r\theta}{t} = r(angularspeed)$$