The Unit Circle

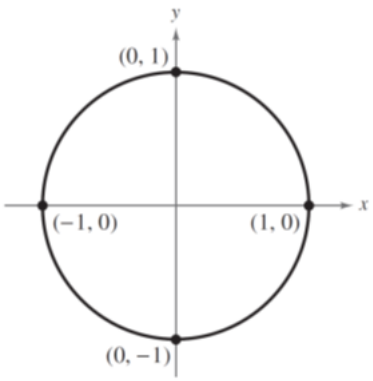
The two historical perspectives of trigonometry incorporate different methods of introducing the trigonometric functions.

Our first introduction to these functions is based on the unit circle.

Consider the **unit circle** given by

$$x^2 + y^2 = 1$$ Unit circle

as shown in Figure 4.18.



Unit Circle

Definitions of Trigonometric Functions

Let t be a real number and let (x, y) be the point on the unit circle corresponding to t.

$\sin t = y$ $\quad \cos t = x$ $\quad \tan t = \frac{y}{x}, \quad x \neq 0$

$\csc t = \frac{1}{y}, \quad y \neq 0$ $\quad \sec t = \frac{1}{x}, \quad x \neq 0$ $\quad \cot t = \frac{x}{y}, \quad y \neq 0$

$$\cos\left(-\frac{\pi}{3}\right)$$

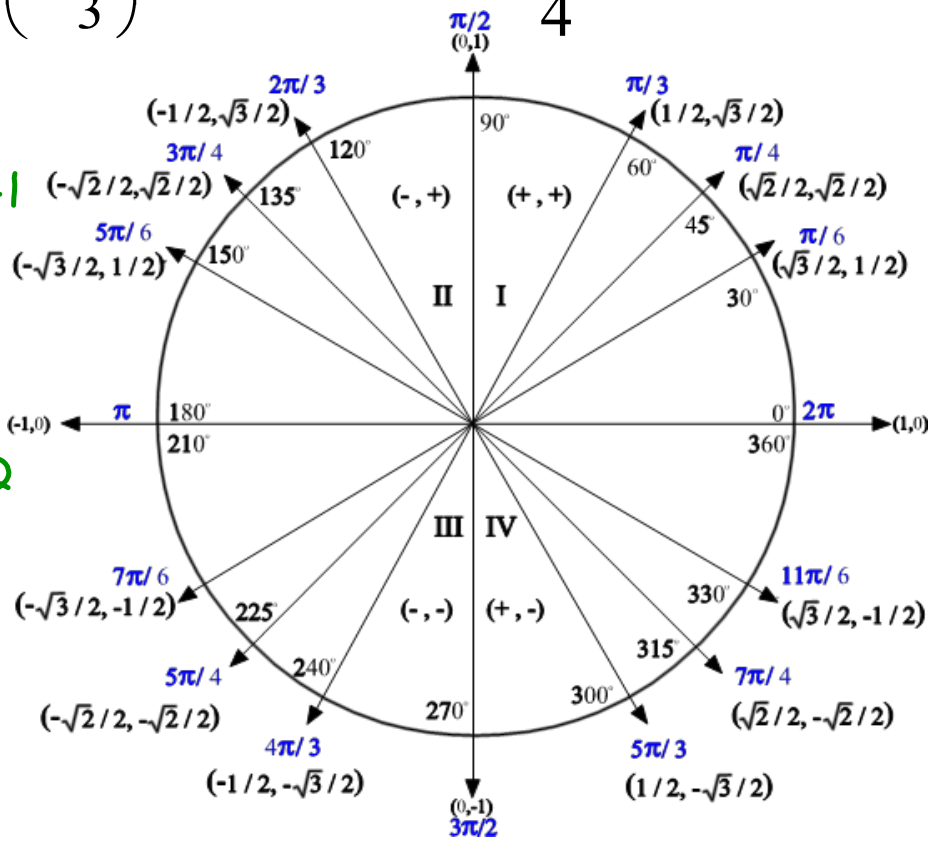
$$\tan\frac{3\pi}{4} = -1$$

$$\csc\frac{7\pi}{6} = -2$$

$\frac{\sqrt{2}}{2} = -1$

$-\frac{\sqrt{2}}{2}$

$\frac{1}{-\frac{1}{2}} = -2$



Sine and Cosine

1 cycle

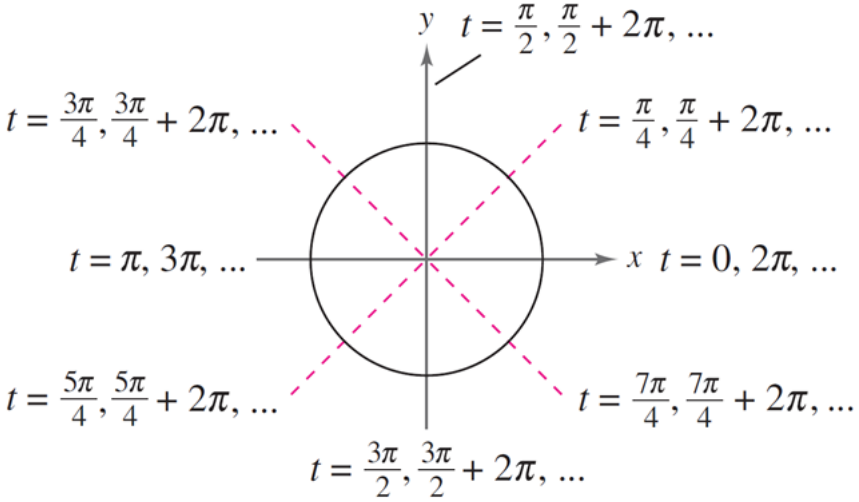
Sinθ: Domain $[0, 2\pi]$ Range $[-1, 1]$

Cosθ: Domain $[0, 2\pi]$ Range $[-1, 1]$

Domain	set of all real numbers $(-\infty, \infty)$
Range	$[-1, 1]$
Period (how long does it take until it starts to repeat)	2π

Domain and Period of Sine and Cosine

Adding 2π to each value of in the interval $[0, 2\pi]$ completes a second revolution around the unit circle, as shown in Figure 4.23.



Domain and Period of Sine and Cosine

The values of $\sin(t + 2\pi)$ and $\cos(t + 2\pi)$ correspond to those of $\sin t$ and $\cos t$.

Similar results can be obtained for repeated revolutions (positive or negative) around the unit circle. This leads to the general result

$$\sin(t + 2\pi n) = \sin t \quad \text{and} \quad \cos(t + 2\pi n) = \cos t$$

for any integer n and real number t. Functions that behave in such a repetitive (or cyclic) manner are called **periodic.**

along the same lines as coterminal

Domain and Period of Sine and Cosine

A function f is *even* when

$$f(-t) = f(t)$$

and is *odd* when

$$f(-t) = -f(t)$$

Of the six trigonometric functions, two are even and four

Even and Odd Trigonometric Functions

The cosine and secant functions are *even*.

$\cos(-t) = \cos t$ $\sec(-t) = \sec t$

The sine, cosecant, tangent, and cotangent functions are *odd*.

$\sin(-t) = -\sin t$ $\csc(-t) = -\csc t$

$\tan(-t) = -\tan t$ $\cot(-t) = -\cot t$

Evaluate **without** a calculator

$\sin\frac{31\pi}{6}$ $= -\frac{1}{2}$ $\frac{31\pi}{6} - \frac{12\pi}{6} = \frac{19\pi}{6} - \frac{12\pi}{6} = \frac{7\pi}{6}$

Evaluate **with** a calculator

Must be in radian mode!

$\tan\frac{4\pi}{3}$ $= 1.73$

$\cos 3$ $= -0.99$

$\csc\frac{2\pi}{7}$ $\frac{1}{\sin\frac{2\pi}{7}} = 1.279$

***make sure to use 1/trig function or can use x^{-1} not $\sin^{-1}$

Definition of Periodic Function

A function f is **periodic** when there exists a positive real number c such that

$$f(t + c) = f(t)$$

for all t in the domain of f. The least number c for which f is periodic is called the **period** of f.

- coterminal to 165^0 (one pos. one neg.)
- convert to radians 132^0
- convert to degrees $\frac{3\pi}{8}$

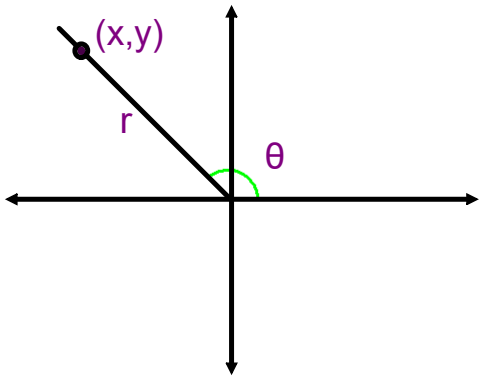
Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{a^2 + b^2}$

$\sin\theta = \frac{y}{r}$ $\csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\sec\theta = \frac{r}{x}$

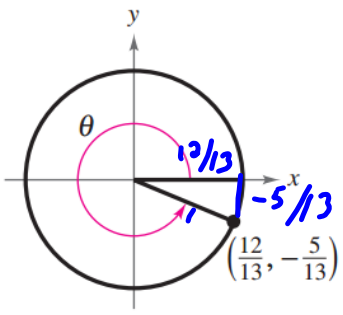
$\tan\theta = \frac{y}{x}$ $\cot\theta = \frac{x}{y}$



Model Homework Problems

Determining Values of Trigonometric Functions In Exercises 9–12, determine the exact values of the six trigonometric functions of the angle θ.

11.



$\frac{-5}{13} \div \frac{12}{13}$

$\frac{-5}{13} \cdot \frac{13}{12}$

$\frac{-5}{12}$

$(12/13)^2 + (-5/13)^2 = 1$

$\sin\theta = 5/13$ $\quad \csc\theta = \frac{13}{-5}$

$\cos\theta = 12/13$ $\quad \sec\theta = 13/12$

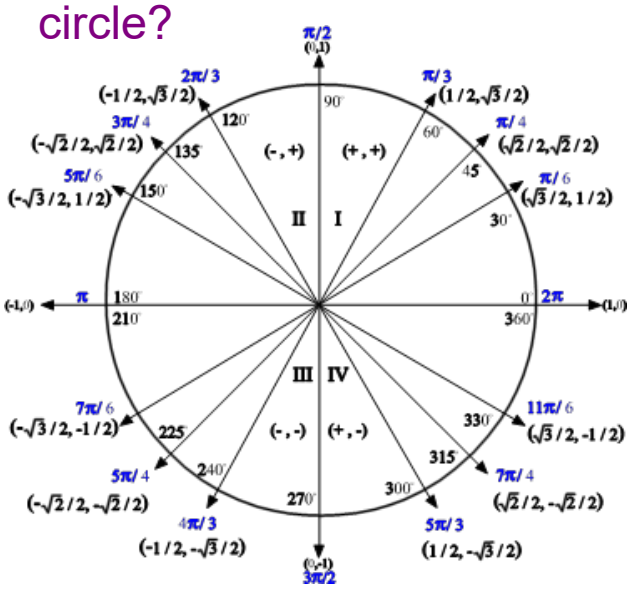
$\tan\theta = \frac{-5}{12}$ $\quad \cot\theta = -\frac{12}{5}$

Model Homework Problems

Evaluating Sine, Cosine, and Tangent **In Exercises 23–32, evaluate (if possible) the sine, cosine, and tangent of the real number.**

29. $t = -\frac{5\pi}{3}$

Where is this on the unit circle?



$\sin\theta$

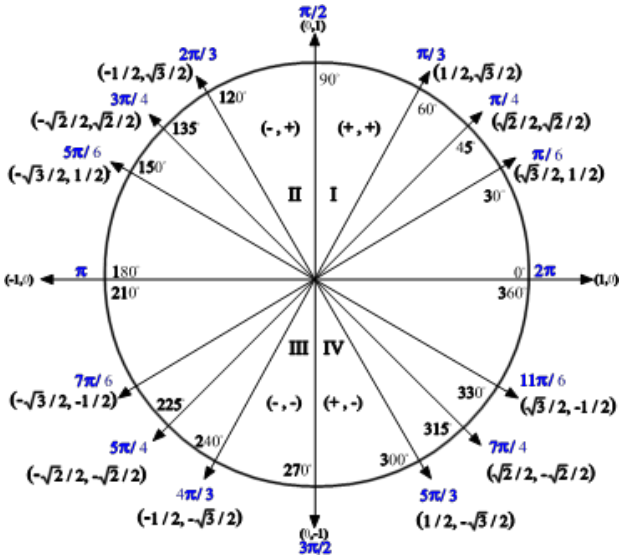
$\cos\theta$

$\tan\theta$

Model Homework Problems

Using the Period to Evaluate Sine and Cosine **In Exercises 39–46, evaluate the trigonometric function using its period as an aid.**

45. $\sin\left(-\frac{9\pi}{4}\right)$

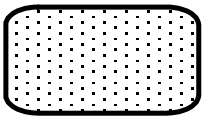


Model Homework Problems

Using a Calculator **In Exercises 53–70, use a calculator to evaluate the trigonometric expression. Round your answer to four decimal places.**

61. csc 0.8

$$\frac{1}{\sin(0.8)}$$



https://create.kahoot.it/details/49ad3232-f188-41da-8dd2-b29b8640572c

https://create.kahoot.it/details/duplicate-of-unit-circle/31cec6a5-84fd-4817-a110-c0a25cc8d4c2