1. Find the values of the six trigonometric functions for angle θ in standard position if a point with the given coordinates lies on its terminal side.

a. (8,5)

b. (-5,-9)

Suppose θ is an angle in standard position whose terminal side lies in the given quadrant. For each function, find the values of the remaining five trigonometric functions of θ.

a. $\cos\theta = \frac{7}{12}$

b. $\tan\theta = \frac{4}{3}$

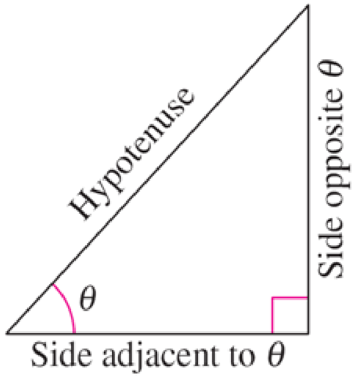
Draw the angle in standard position

a. 60^o

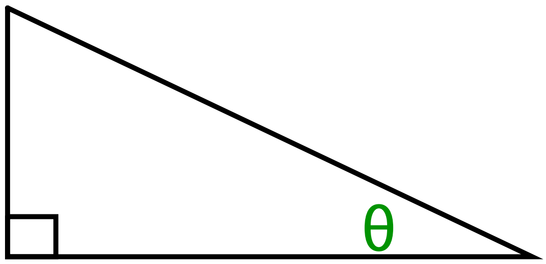
b. 780^o

c. -630^o

The Six Trigonometric Functions



Remember soh - cah - toa



opposite adjacent hypotenuse

soh - cah - toa

$\sin\theta = \frac{opp}{hyp}$ | $\cos\theta = \frac{adj}{hyp}$ | $\tan\theta = \frac{opp}{adj}$

What are the reciprocals of sine, cosine and tangent?

$\csc\theta =$

$\sec\theta =$

$\cot\theta =$

The Six Trigonometric Functions

Right Triangle Definitions of Trigonometric Functions

Let θ be an *acute* angle of a right triangle. Then the six trigonometric functions *of the angle* θ are defined as follows. (Note that the functions in the second column are the *reciprocals* of the corresponding functions in the first column.)

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$

$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

The abbreviations

"opp," "adj," and "hyp"

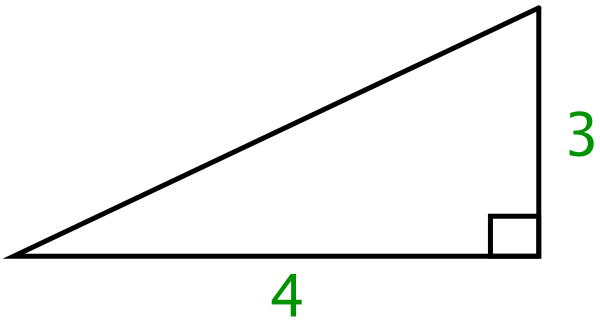
represent the lengths of the three sides of a right triangle.

opp = the length of the side *opposite* θ

adj = the length of the side *adjacent* to θ

hyp = the length of the *hypotenuse*

Find exact values



$\sin\theta =$ $\csc\theta =$

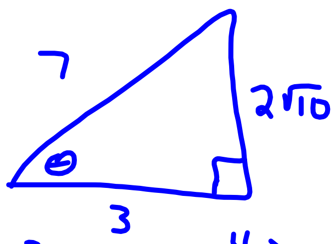
$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

With a partner: One page per group. You will hand in at the end of the period.

Sketch triangle and find the other trig. functions

1. $\cos\theta = \frac{3}{7}$



$7^2 - 3^2 = b^2$
$49 - 9 = b^2$
$\sqrt{40} = b$

40
4 10
2 2 2 5

$\sin\theta = \frac{2\sqrt{10}}{7}$

$\cos\theta = \frac{3}{7}$

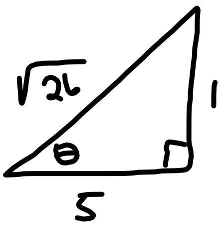
$\tan\theta = \frac{2\sqrt{10}}{3}$

$\csc\theta = \frac{7}{2\sqrt{10}} = \frac{7\sqrt{10}}{20}$

$\sec\theta = \frac{7}{3}$

$\cot\theta = \frac{3}{2\sqrt{10}} = \frac{3\sqrt{10}}{20}$

2. $\cot\theta = \frac{5}{1}$



$5^2 + 1^2 = c^2$

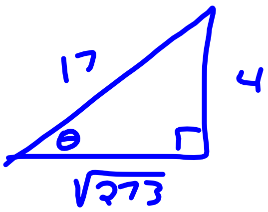
$c = \sqrt{26}$

$\sin\theta = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$ $\csc\theta = \frac{\sqrt{26}}{1}$

$\cos\theta = \frac{5}{\sqrt{26}} = \frac{5\sqrt{26}}{26}$ $\sec\theta = \frac{\sqrt{26}}{5}$

$\tan\theta = \frac{1}{5}$ $\cot\theta = \frac{5}{1}$

3. $\csc\theta = \frac{17}{4}$



$17^2 - 4^2$

$\sin\theta = \frac{4}{17}$ $\csc\theta = \frac{17}{4}$

$\cos\theta = \frac{\sqrt{273}}{17}$ $\sec\theta = \frac{17}{\sqrt{273}} = \frac{17\sqrt{273}}{273}$

$\tan\theta = \frac{4}{\sqrt{273}} = \frac{4\sqrt{273}}{273}$ $\cot\theta = \frac{\sqrt{273}}{4}$

The Six Trigonometric Functions

Sines, Cosines, and Tangents of Special Angles

$\sin 30° = \sin \frac{\pi}{6} = \frac{1}{2}$ $\quad \cos 30° = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$ $\quad \tan 30° = \tan \frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\sin 45° = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \cos 45° = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\quad \tan 45° = \tan \frac{\pi}{4} = 1$

$\sin 60° = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ $\quad \cos 60° = \cos \frac{\pi}{3} = \frac{1}{2}$ $\quad \tan 60° = \tan \frac{\pi}{3} = \sqrt{3}$

In the box, note that $\sin 30° = \frac{1}{2} = \cos 60°$. This occurs because 30° and 60° are complementary angles, and, in general, it can be shown from the right triangle definitions that *cofunctions of complementary angles are equal*.

Trigonometric Identities

p. 270 We already know Reciprocal Identities
Quotient Identities

now presented more formally

We will get to Pythagorean Identities

HUGE!!!!

Chapter 5

Trigonometric Identities

In trigonometry, a great deal of time is spent studying relationships between trigonometric functions (identities).

Fundamental Trigonometric Identities

Reciprocal Identities

$\sin\theta = \dfrac{1}{\csc\theta}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta}$

$\cos\theta = \dfrac{1}{\sec\theta}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta}$

$\tan\theta = \dfrac{1}{\cot\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta}$

Quotient Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$$\sin^2\theta + \cos^2\theta = 1$$

$$1 + \tan^2\theta = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

A little preview

Quotient identities: already know...

$$\text{tangent} = \frac{y}{x} = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{sine}}{\text{cosine}}$$

show $\cos\theta\sec\theta = 1$

$$\cos\theta \cdot \frac{1}{\cos\theta} = 1$$

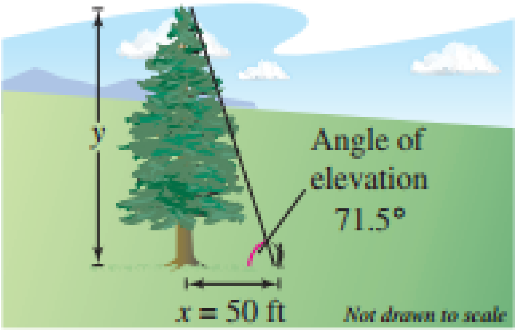
$$1 = 1$$

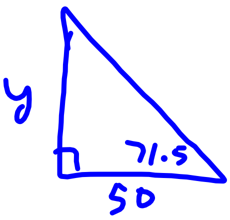
Applications

Many applications of trigonometry involve a process called **solving right triangles.**

In this type of application, you are usually given one side of a right triangle and one of the acute angles and are asked to find one of the other sides, *or* you are given two sides and are asked to find one of the acute angles.

4. A surveyor is standing 50 feet from the base of a large tree, as shown in Figure 4.31. The surveyor measures the angle of elevation to the top of the tree as 71.5°. How tall is the tree?





$y = 149.4$ ft

$\tan 71.5 = \frac{y}{50}$

$50 \tan 71.5 = y$

5.
You are 200 yards from a river. Rather than walking directly to the river, you walk 400 yards along a straight path to the river's edge. Find the acute angle θ between this path and the river's edge, as illustrated in Figure 4.32.

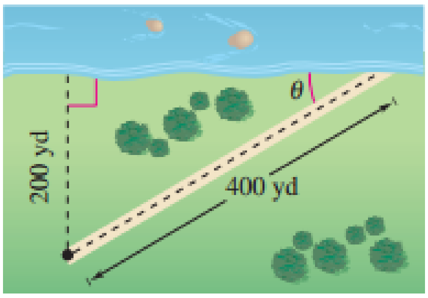
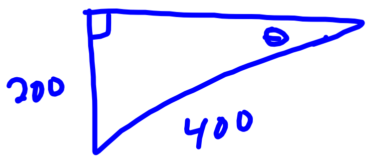


Figure 4.32



$\sin\theta = \frac{200}{400}$

$\sin\theta = \frac{1}{2}$

$\sin^{-1}(1/2)$

$\theta = 30^\circ$

6.

Find the length c of the skateboard ramp shown in Figure 4.33.

$\sin 18.4 = \frac{4}{c}$

$c = 4/\sin 18.4$

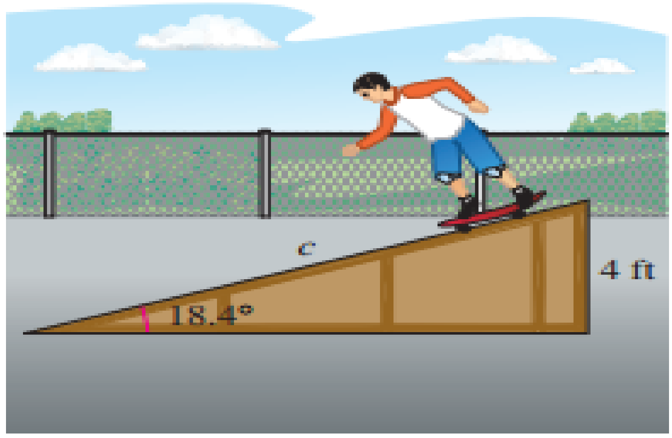
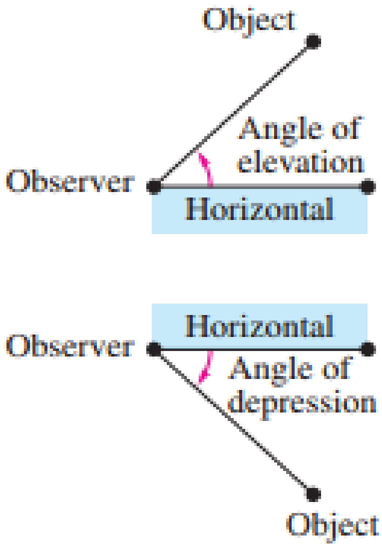
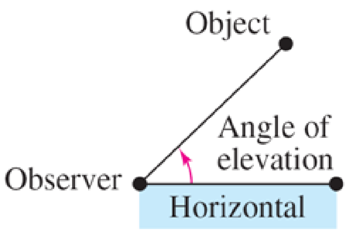


Figure 4.33


Object
Angle of
elevation
Observer
Horizontal
Horizontal
Observer
Angle of
depression
Object

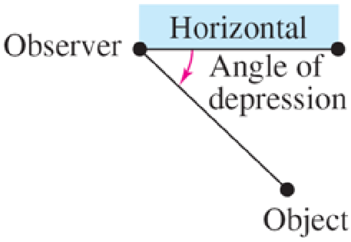
Applications

The angle you are given is the **angle of elevation**, which represents the angle from the horizontal upward to the object.



Applications

In other applications you may be given the **angle of depression,** which represents the angle from the horizontal downward to the object.



Model Homework Problems

Using a Calculator **In Exercises 31–36, use a calculator to evaluate each function. Round your answers to four decimal places. (Be sure the calculator is in the correct angle mode.)**

33. (a) $\sec 42° \, 12'$

(b) $\csc 48° \, 7'$