What You Should Learn

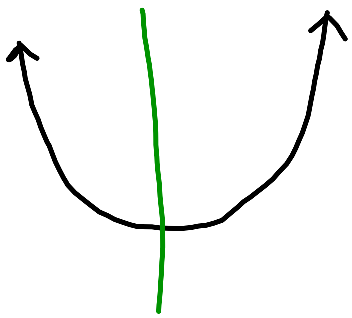
- Evaluate and graph inverse sine functions
- Evaluate and graph other inverse trigonometric functions
- Evaluate compositions of trigonometric functions

Inverse Sine Function

One-to-one

What does that mean?

A function must be one-to-one to have an inverse.

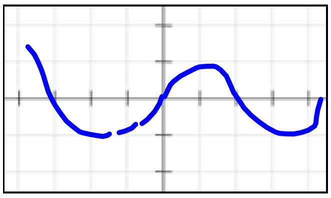


One-to-one?

Can I make it one to one?

Let $f(x) = \sin x$.

a. Graph f on the interval $[-2\pi, 2\pi]$.

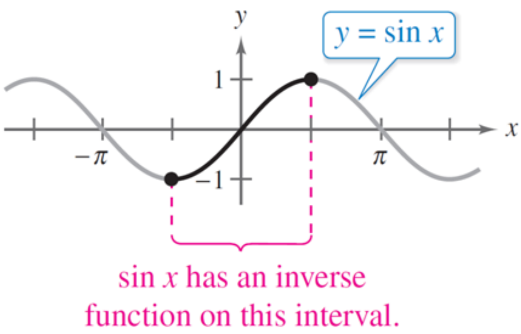


b. Is $f(x) = \sin x$ a one-to-one function? ________? Why or Why not? ______________________________

Therefore, we must restrict the domain of f to make it one-to-one. We want to restrict the domain as close to the origin as possible. We restrict the domain of f to $-\pi/2$ to $\frac{\pi}{2}$ to make it a one-to-one function.

it is obvious that y = sin x does not pass the

test because different values of x yield the same y-value.



However, when you restrict the domain to the interval

$-\pi/2 \le x \le \pi/2$ (corresponding to the black portion)

1. On the interval $[-\pi/2, \pi/2]$, the function $y = \sin x$ is increasing.

2. On the interval $[-\pi/2, \pi/2]$, $y = \sin x$ takes on its full range of values, $-1 \le \sin x \le 1$.

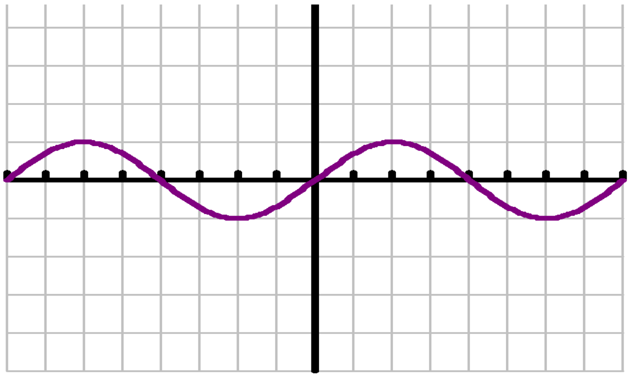
3. On the interval $[-\pi/2, \pi/2]$, $y = \sin x$ is one-to-one.

$y = \arcsin x$ or $y = \sin^{-1} x$.

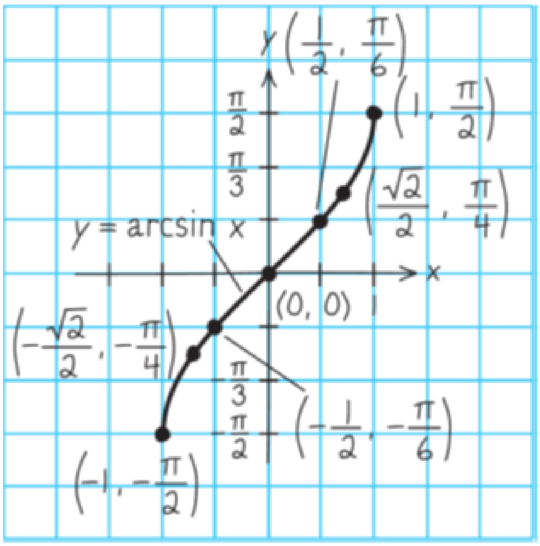
The notation $\sin^{-1} x$ is consistent with the inverse function notation $f^{-1}(x)$. The arcsin x notation (read as "the arcsine of x") comes from the association of a central angle with its intercepted *arc length* on a unit circle.

So, arcsin x means the angle (or arc) whose sine is x. Both notations, arcsin x and $\sin^{-1} x$, are commonly used in mathematics, so remember that $\sin^{-1} x$ denotes the *inverse* sine function rather than $1/\sin x$. The values of arcsin x lie in the interval $-\pi/2 \leq \arcsin x \leq \pi/2$.

y =sin *x*



Need to restrict domain so it remains a function



Inverse Functions:
switch domain and range (x and y)

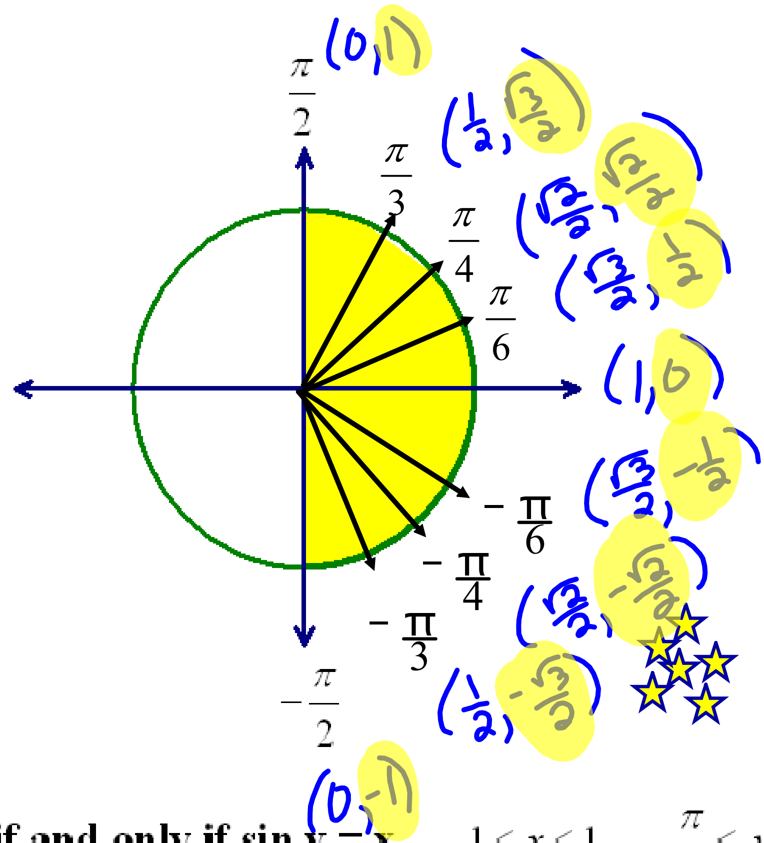
can put in your calculator
$y=\sin^{-1}x$

Change window for your x and y.

D: [-1, 1] R: $[-\frac{\pi}{2}, \frac{\pi}{2}]$

arcsin

$\sin^{-1}$



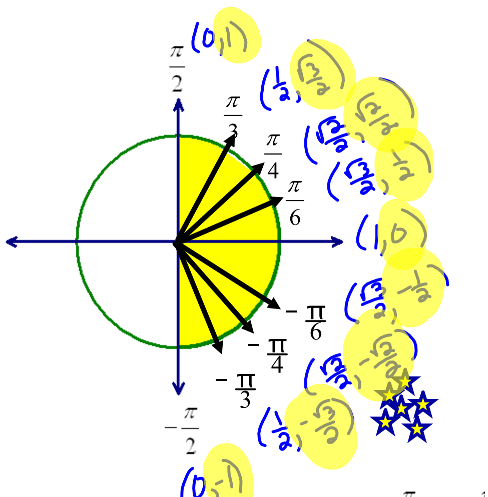
y = arcsin(x) if and only if sin y = x $\quad -1 \le x \le 1 \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

1. $\arcsin(\frac{1}{2})$ $\frac{\pi}{6}$

2. $\sin^{-1}(\frac{1}{2})$ $\frac{\pi}{6}$

3. $\arcsin(-\frac{1}{2})$ $-\frac{\pi}{6}$

4. $\arcsin(-\frac{\sqrt{2}}{2})$ $-\frac{\pi}{4}$



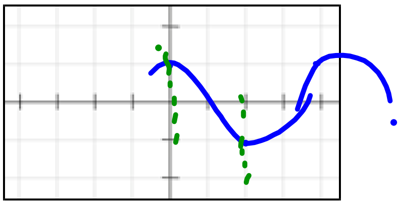
y = arcsin(x) if and only if sin y = x $-1 \le x \le 1$ $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

5. $\sin^{-1}(2)$

The Inverse Cosine Function

Let $f(x) = \cos x$.

a. Graph f on the interval $[-2\pi, 2\pi]$



b. Since $f(x) = \cos x$ is not a one-to-one function we must restrict the domain of f to make it one-to-one.

Therefore, we restrict the domain of f to __0 to π__ to make it a one-to-one function.

Other Inverse Trigonometric Functions

The cosine function is decreasing and one-to-one on the interval $0 \le x \le \pi$, as shown in Figure 4.69.

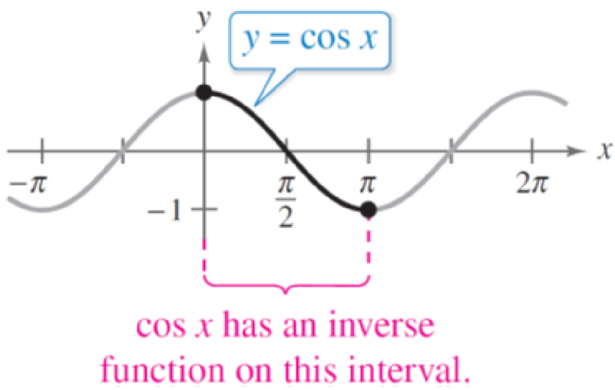
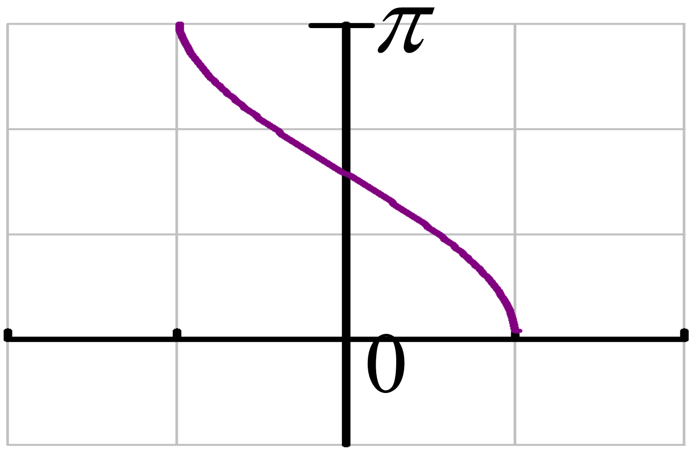


Figure 4.69

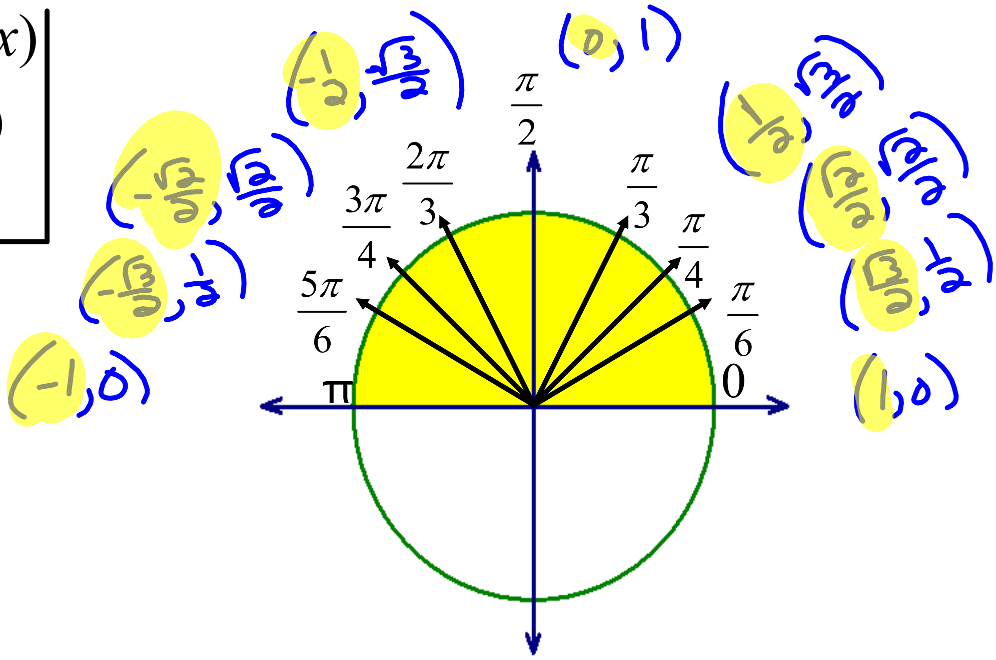
Other Trig Functions--p. 311

$\cos^{-1}(x)$
$\arccos(x)$



arccos (x)

$\cos^{-1}(x)$



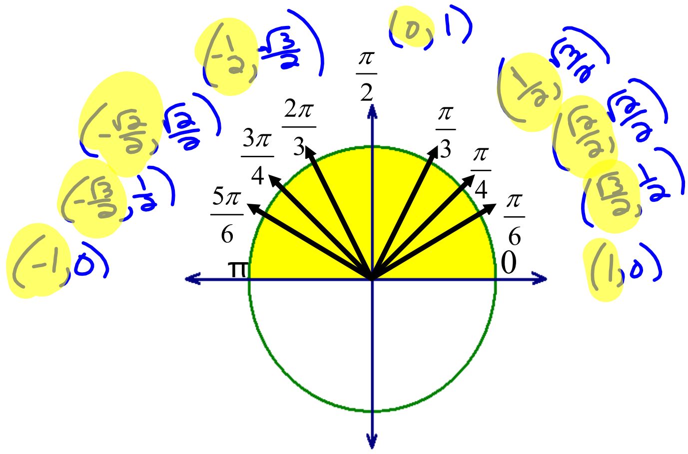
y = arcos(x) if and only if cos y = x $-1 \le x \le 1 \quad 0 \le y \le \pi$

1. $\arccos(\frac{1}{2})$ $\frac{\pi}{3}$

2. $\cos^{-1}(-\frac{\sqrt{3}}{2})$ $\frac{5\pi}{6}$

3. $\arccos(-1)$ π

4. $\cos^{-1}(0)$ $\frac{\pi}{2}$

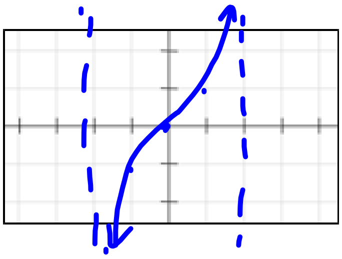


y = arcos(x) if and only if cos y = x $-1 \le x \le 1$ $0 \le y \le \pi$

The Inverse Tangent Function

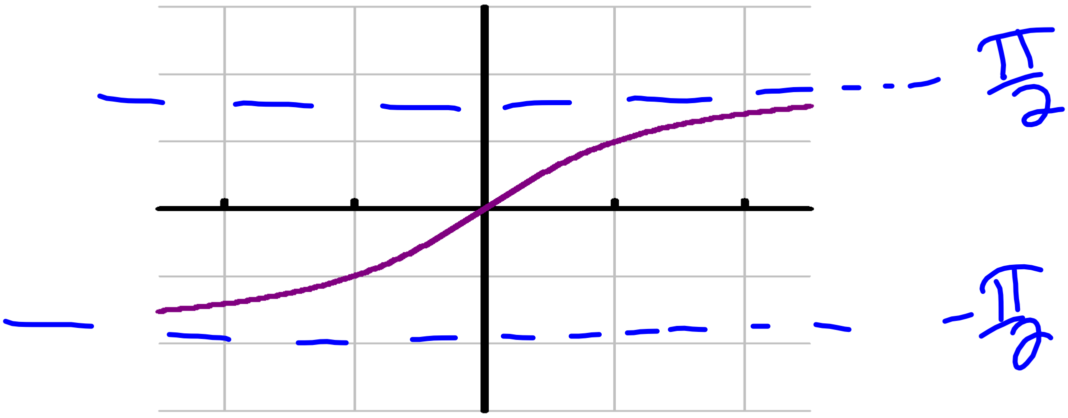
Let $f(x) = \tan x$.

a. Graph f on the interval $[-2\pi, 2\pi]$.



d. Since $f(x) = \tan x$ is not a one-to-one function we must restrict the domain of f to make it one-to-one. Therefore, we restrict the domain of f to $-\pi/2$ to $\pi/2$ to make it a one-to-one function.

tan $^{-1}$(x) a.k.a. arctan (x)

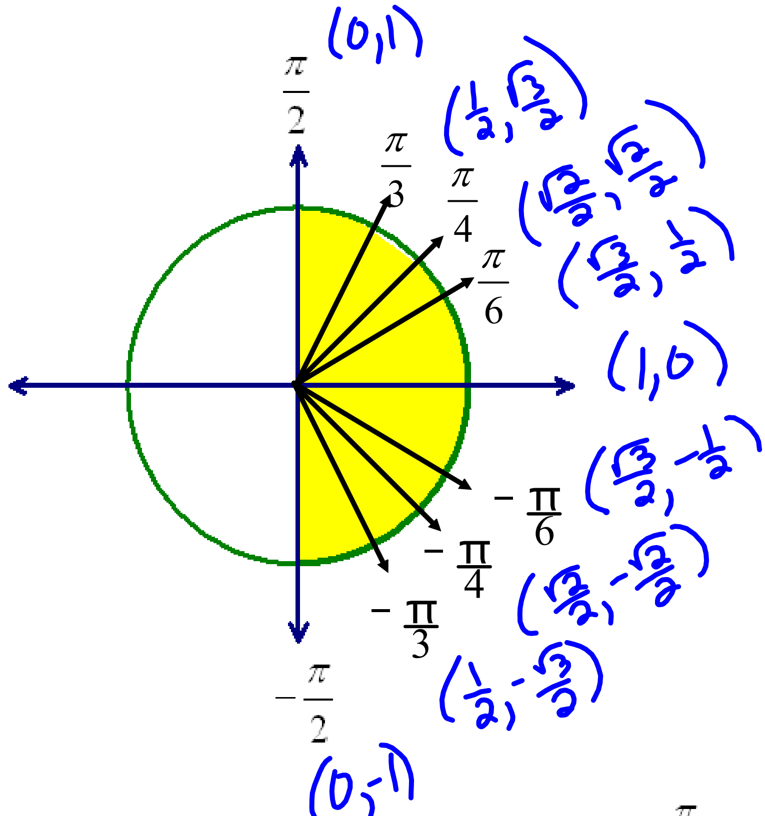


D: $(-\infty, +\infty)$

R: $(-\pi/2, \pi/2)$

Function	*Domain*	*Range*
arcsin $\sin^{-1}$	$-1 \le x \le 1$	$\frac{-\pi}{2} \le y \le \frac{\pi}{2}$
arccos $\cos^{-1}$	$-1 \le x \le 1$	$0 \le y \le \pi$
arctan $\tan^{-1}$	$-\infty \le x \le \infty$	$\frac{-\pi}{2} < y < \frac{\pi}{2}$

arcsin (x)
arctan (x)



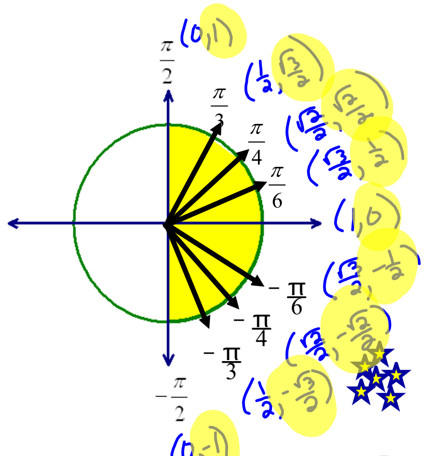
y = arcsin(x) if and only if sin y = x $\quad -1 \le x \le 1 \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$

1. $\arctan(1)$ $\frac{\pi}{4}$

2. $\tan^{-1}(0)$ 0

3. $\arctan(\frac{\sqrt{3}}{3})$ $\frac{\pi}{6}$

4. $\arctan(-\frac{\sqrt{3}}{3})$ $-\frac{\pi}{6}$



y = arcsin(x) if and only if sin y = x $-1 \le x \le 1$ $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Compositions of Functions

$f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

Inverse Properties

If $-1 \leq x \leq 1$ and $-\pi/2 \leq y \leq \pi/2$, then

$\sin(\arcsin x) = x$ and $\arcsin(\sin y) = y$.

If $-1 \leq x \leq 1$ and $0 \leq y \leq \pi$, then

$\cos(\arccos x) = x$ and $\arccos(\cos y) = y$.

If x is a real number and $-\pi/2 < y < \pi/2$, then

$\tan(\arctan x) = x$ and $\arctan(\tan y) = y$.

Keep in mind that these inverse properties do not apply for arbitrary values of *x* and *y*. For instance,

$$\arcsin\left(\sin\frac{3\pi}{2}\right) = \arcsin \quad = \quad$$

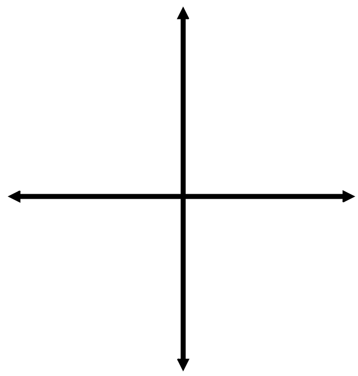
ex. tan(arctan (- 5)

arcsin $\left(\sin \frac{5\pi}{3}\right)$

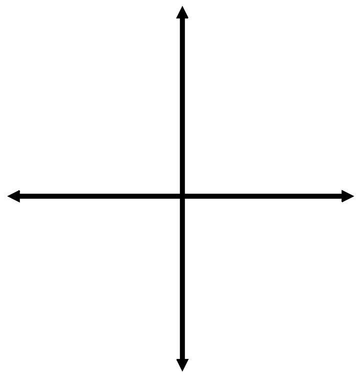
not in range
need to rewrite

ex.

$$\tan(\arccos\frac{2}{3})$$



$$\cos(\arcsin(-\frac{3}{5}))$$



☆ sin(arccos 3x)