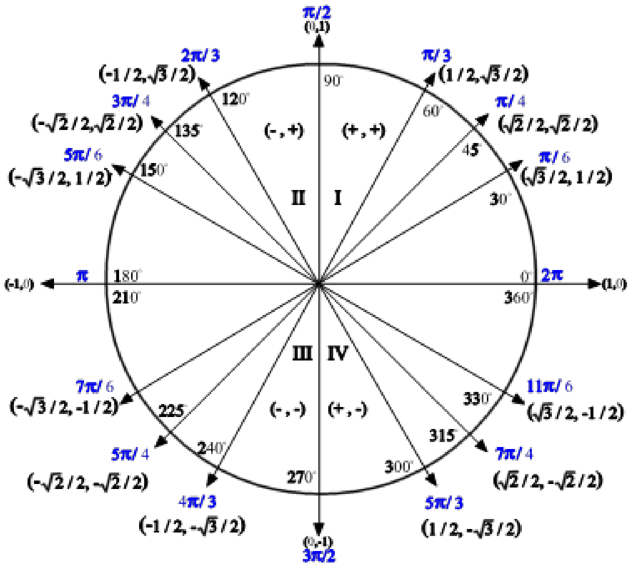
5.3 Solving Trigonometric Functions

Warm Ups... Unit Circle Review...

$\sin x = \frac{\sqrt{3}}{2}$ $\frac{\pi}{3}, \frac{2\pi}{3}$ $\cos x = \frac{\sqrt{2}}{2}$

$\cos x = \frac{1}{2}$ $\tan x = \sqrt{3}$

$\sec x = 2$ $\tan x = 0$



Period of sin / cos = Period of tan =

To solve a trigonometric equation...
use standard algebraic techniques such as collecting like terms and factoring (5.1-5.2)

The preliminary goal in solving trigonometric equations is...
isolate the trigonometric function

Solve: $2 \sin x - 1 = 0$

$\frac{2\sin x}{2} = \frac{1}{2}$

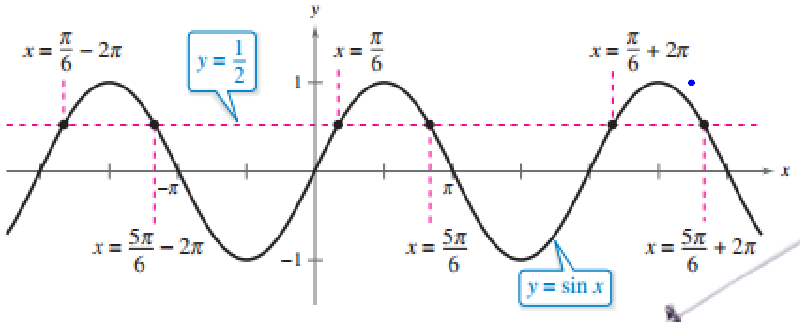
$\sin x = \frac{1}{2}$

$[0, 2\pi)$

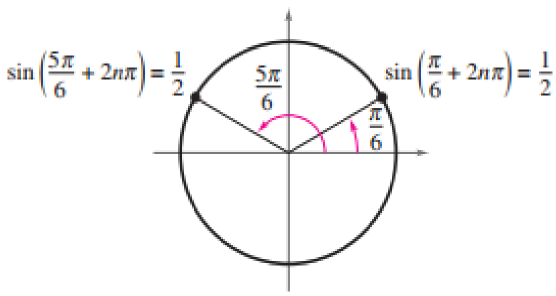
$\frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{6} + 2\pi n$

$x = \frac{5\pi}{6} + 2\pi n$



Another way to show that the equation $\sin x = \frac{1}{2}$ has infinitely many solutions is indicated in Figure 5.5. Any angles that are coterminal with $\pi/6$ or $5\pi/6$ are also solutions of the equation.



$$4\sin^2 x - 3 = 0$$

$$\frac{4\sin^2 x}{4} = \frac{3}{4}$$

$$\sqrt{\sin^2 x} = \sqrt{\frac{3}{4}}$$

$$\sin x = \frac{\pm\sqrt{3}}{\sqrt{4}}$$

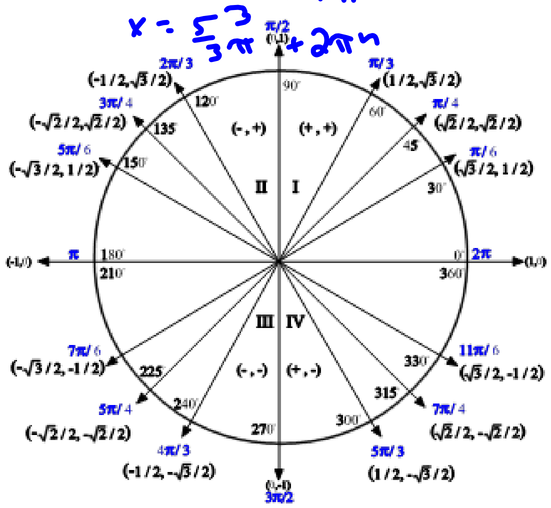
$$\sin x = \pm\frac{\sqrt{3}}{2}$$

$$x = \frac{\pi}{3} + 2\pi n$$

$$x = \frac{2\pi}{3} + 2\pi n$$

$$x = \frac{4\pi}{3} + 2\pi n$$

$$x = \frac{5}{3}\pi + 2\pi n$$



Solve on [0, 2π]

$$\cos x - \sqrt{2} = -\cos x$$

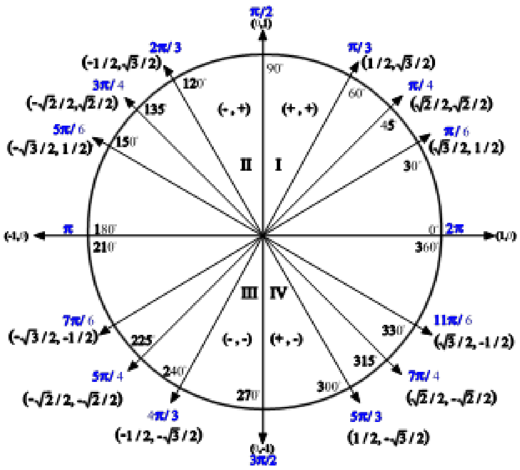
$+\cos x$ $\quad\quad$ $+\cos x$

$2\cos x - \sqrt{2} = 0$

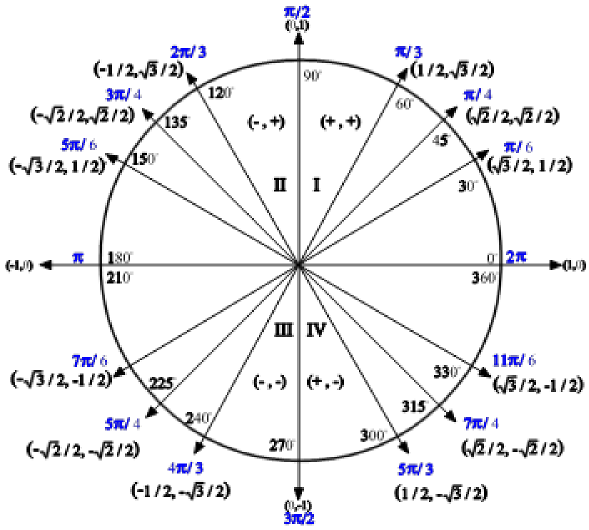
$\frac{2\cos x}{2} = \frac{\sqrt{2}}{2}$

$\cos x = \frac{\sqrt{2}}{2}$

$x: \frac{\pi}{4}, \frac{7\pi}{4}$



$$\sin^2 x = 2\sin x$$


π/2
(0,1)
2π/3
(-1/2,√3/2)
π/3
(1/2,√3/2)
3π/4
(-√2/2,√2/2)
π/4
(√2/2,√2/2)
5π/6
(-√3/2,1/2)
π/6
(√3/2,1/2)
90°
120°
60°
135
45
150°
30°
(-,+)
(+,+)
II
I
(-1,0)
π
180°
210°
0°
2π
360°
(1,0)
III
IV
7π/6
(-√3/2,-1/2)
11π/6
(√3/2,-1/2)
225
(-,-)
(+,-)
330°
315
5π/4
(-√2/2,-√2/2)
7π/4
(√2/2,-√2/2)
240°
270°
300°
4π/3
(-1/2,-√3/2)
5π/3
(1/2,-√3/2)
(0,-1)
3π/2

$$2\sin^2 x + 3\cos x - 3 = 0$$

) to

be x ... general

solution is

$$x = 2n\pi, \quad x = \frac{\pi}{3} + 2n\pi, \quad x = \frac{5\pi}{3} + 2n\pi$$ General solution

Find all solutions of $\cos x + 1 = \sin x$ in the interval $[0, 2\pi)$.

Solution

It is not clear how to rewrite this equation in terms of a single trigonometric functic
Notice what happens when you square each side of the equation.

Write original equation.

Square each side.

Pythagorean identity

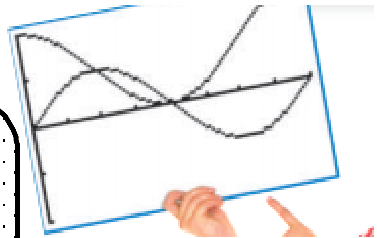
Combine like terms.

Factor.

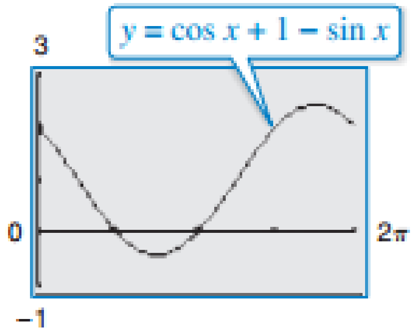
Setting each factor equal to zero produces the following.

Because you squared the original equation, check for extraneous solutions.

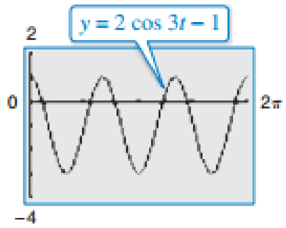
Check



Of the three possible solutions, $x = 3\pi/2$ is extraneous. So, in the interval $[0, 2\pi)$, the only solutions are $x = \pi/2$ and $x = \pi$. The graph of $y = \cos x + 1 - \sin x$, shown in Figure 5.11, confirms this result because the graph has two x-intercepts (at $x = \pi/2$ and $x = \pi$) in the interval $[0, 2\pi)$.



$$2\cos 3t - 1 = 0$$



Solve: $3\tan^2 x - 1 = 0$

Solve

$$\tan^2 x + 2\tan x = -1$$

Solve on $\left[0, 2\pi\right]$

$$2\sin^2 x - \sin x - 1 = 0$$

$$3\tan\frac{x}{2}+3=0$$

In the interval $[0, \pi)$, you know that $x/2 = 3\pi/4$ is the only solution. So in general, you have $x/2 = 3\pi/4 + n\pi$. Multiplying this result by 2, you obtain the general solution

where n is an integer. This solution is confirmed graphically in Figure 5.13.

Solve $\sec^2 x - 2\tan x = 4$.

Solution

Write original equation.

Pythagorean identity

Combine like terms.

Factor.

Setting each factor equal to zero, you obtain two solutions in the interval $(-\pi/2, \pi/2)$. [Recall that the range of the inverse tangent function is $(-\pi/2, \pi/2)$.]

and

Finally, because $\tan x$ has a period of π, add multiples of π to obtain

and General solution

where n is an integer. This solution is confirmed graphically in Figure 5.14.

Approximate the solutions of

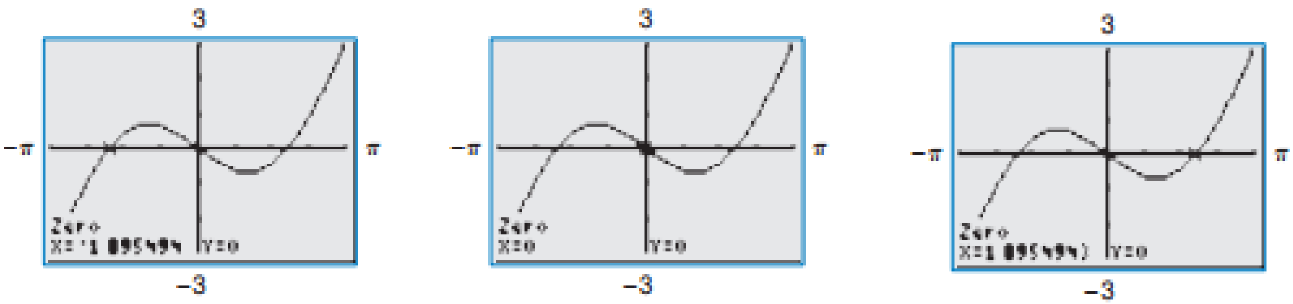
$$x = 2 \sin x$$

in the interval $[-\pi, \pi]$.

Use a graphing utility to graph $y = x - 2 \sin x$ in the interval $[-\pi, \pi]$. Using the *zero* or *root* feature, you can see that the solutions are

$$x \approx -1.8955, \quad x = 0, \quad \text{and} \quad x \approx 1.8955.$$

See Figure 5.15.



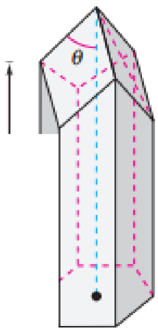
The surface area of a honeycomb is given by the equation

$$S = 6hs + \frac{3}{2}s^2\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right), \qquad 0 < \theta \le 90^\circ$$

where $h = 2.4$ inches, $s = 0.75$ inch, and θ is the angle indicated in Figure 5.16.

a. What value of θ gives a surface area of 12 square inches?

b. What value of θ gives the minimum surface area?



a. Let $h = 2.4$, $s = 0.75$, and $S = 12$.

$$S = 6hs + \frac{3}{2}s^2\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right)$$

Using a graphing utility set in *degree* mode, you can graph the function

Using the *zero* or *root* feature, you can determine that

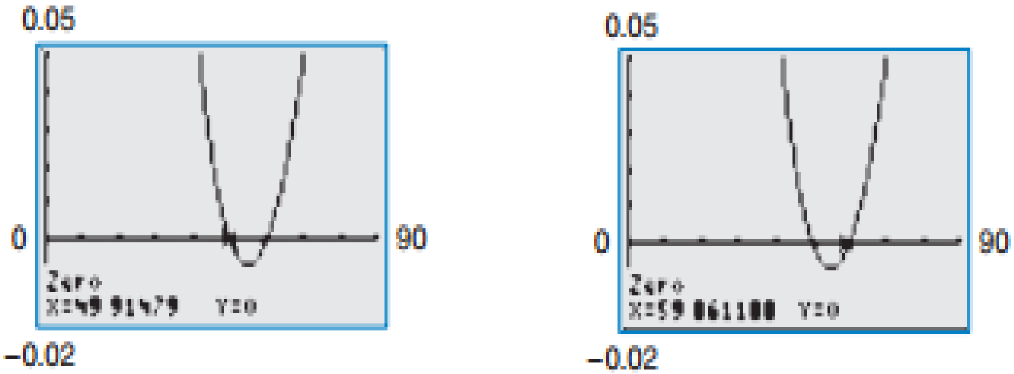


Figure 5.17 $y = 0.84375\left(\frac{\sqrt{3} - \cos x}{\sin x}\right) - 1.2$

From part (a), let $h = 2.4$ and $s = 0.75$ to obtain

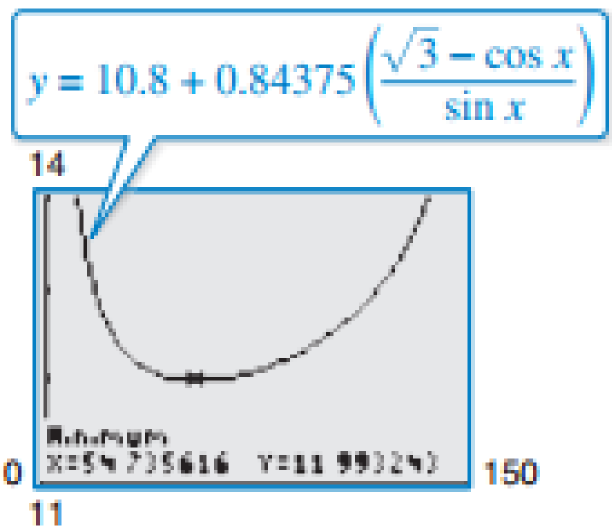
$$S = 10.8 + 0.84375\left(\frac{\sqrt{3} - \cos \theta}{\sin \theta}\right).$$

Graph this function using a graphing utility set in *degree* mode. Use the *minimum* feature to approximate the minimum point on the graph, which occurs at

$$\theta \approx 54.7°$$

as shown in Figure 5.18. By using calculus, it can be shown that the exact minimum value is

$$\theta = \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 54.7356°.$$



Solve

$$2\cos x - \sec x = 0$$

Solve

$$2\sin^2 x + 3\cos x - 3 = 0$$

Summarize the major points

How do you solve trigonometric equations written in quadratic form or containing more than one angle?