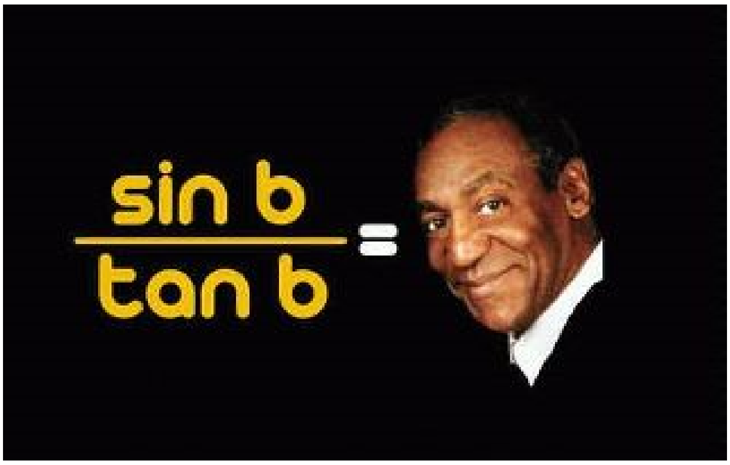
5.3 Day Twooooo....
Solving Multiple Angle Equations
Using a Calculator to Approximate Solutions



Warm Up

1. $\dfrac{1}{\tan^2 x + 1}$ $\cos^2 x$

2. $\dfrac{\sin(-x)\cot x}{\sin\left(\dfrac{\pi}{2} - x\right)}$ -1

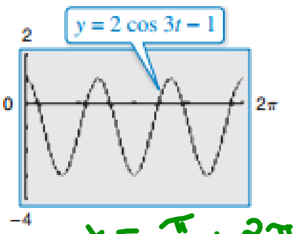
3. $\sin^3 x + \sin x \cos^2 x = \sin x$

4. $\csc^2\left(\dfrac{\pi}{2} - x\right) - 1 = \tan^2 x$

$$2\cos 3t - 1 = 0$$

$$\frac{2\cos 3t}{2} = \frac{1}{2} \qquad \cos 3t = \frac{1}{2}$$

$$\frac{3t}{3} = \frac{\frac{\pi}{3} + n2\pi}{3} \qquad 3t = \frac{5\pi}{3} \;\; \frac{+ n2\pi}{3}$$



$$x = \frac{\pi}{9} + \frac{2\pi}{3}n$$

Solution

$2\cos 3t - 1 = 0$	Write original equation.
$2\cos 3t = 1$	Add 1 to each side.
$\cos 3t = \frac{1}{2}$	Divide each side by 2.

In the interval $[0, 2\pi)$, you know that $3t = \pi/3$ and $3t = 5\pi/3$ are the only solutions. So in general, you have $3t = \pi/3 + 2n\pi$ and $3t = 5\pi/3 + 2n\pi$. Dividing this result by 3, you obtain the general solution

$$t = \frac{\pi}{9} + \frac{2n\pi}{3} \quad \text{and} \quad t = \frac{5\pi}{9} + \frac{2n\pi}{3} \qquad \text{General solution}$$

where n is an integer. This solution is confirmed graphically in Figure 5.12.

Solve: $3\tan^2 x - 1 = 0$

$3\tan^2 x = 1$

$\tan^2 x = \frac{\sqrt{1}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\tan x = \pm \frac{\sqrt{3}}{3}$

$x = \frac{\pi}{6} + n\pi$

$x = \frac{5\pi}{6} + n\pi$

$x = \frac{7\pi}{6} + n\pi$

$x = \frac{11\pi}{6} + n\pi$

$$3\tan\frac{x}{2}+3=0$$

$$3\tan\frac{x}{2} + 3 = 0 \quad \text{Original equation}$$

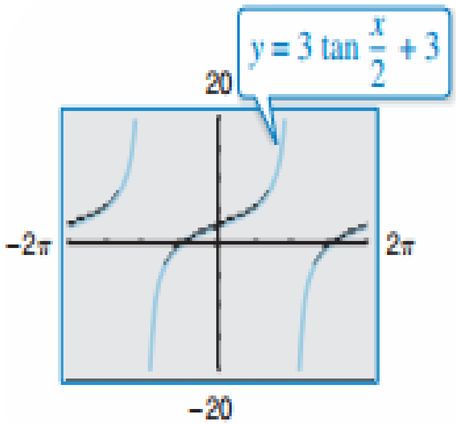
$$3\tan\frac{x}{2} = -3 \quad \text{Subtract 3 from each side.}$$

$$\tan\frac{x}{2} = -1 \quad \text{Divide each side by 3.}$$

In the interval $[0, \pi)$, you know that $x/2 = 3\pi/4$ is the only solution. So in general, you have $x/2 = 3\pi/4 + n\pi$. Multiplying this result by 2, you obtain the general solution

$$x = \frac{3\pi}{2} + 2n\pi \quad \text{General solution}$$

where n is an integer. This solution is confirmed graphically in Figure 5.13.



Solve: $\sin 3x = \frac{\sqrt{2}}{2}$

$$\frac{3x}{3} = \frac{\frac{\pi}{4} + 2\pi n}{3}$$

$$x = \frac{\pi}{12} + \frac{2\pi n}{3}$$

$$3x = \frac{\frac{3\pi}{4}}{3} + \frac{2\pi n}{3}$$

$$x = \frac{3\pi}{12} \quad x = \frac{\pi}{4} + \frac{2\pi n}{3}$$

$3 \tan 2x + 3 = 0$

$$2\cos\left(\frac{x}{2}\right)-1=0$$

Use a graphing utility to approximate the solutions of the equation in the interveral [-π, π)

x=2sinx

Use the graphing calculator to graph

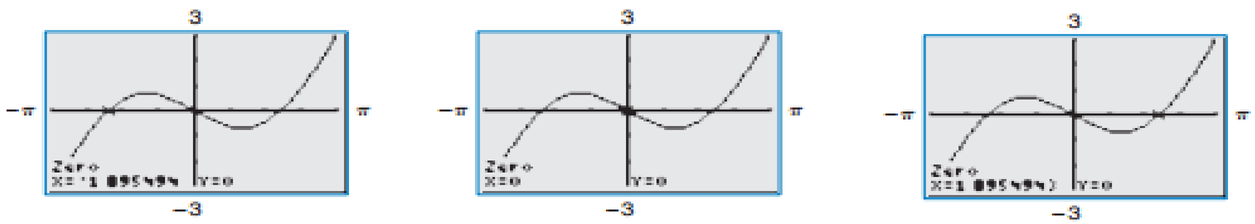
y = x-2sinx

Find the x-intercepts

Solution

Use a graphing utility to graph $y = x - 2\sin x$ in the interval $[-\pi, \pi]$. Using the *zero* or *root* feature, you can see that the solutions are

$x \approx -1.8955,\quad x = 0,\quad \text{and}\quad x \approx 1.8955.$ See Figure 5.15.



$$2\sin^2 x + 3\cos x - 3 = 0$$

Solution

Begin by rewriting the equation so that it has only cosine functions.

$2\sin^2 x + 3\cos x - 3 = 0$	Write original equation.
$2(1 - \cos^2 x) + 3\cos x - 3 = 0$	Pythagorean identity
$2 - 2\cos^2 x + 3\cos x - 3 = 0$	
$2\cos^2 x - 3\cos x + 1 = 0$	Combine like terms and multiply each side by -1.
$(2\cos x - 1)(\cos x - 1) = 0$	Factor.

By setting each factor equal to zero, you can find the solutions in the interval $[0, 2\pi)$ to be $x = 0$, $x = \pi/3$, and $x = 5\pi/3$. Because $\cos x$ has a period of 2π, the general solution is

$$x = 2n\pi, \quad x = \frac{\pi}{3} + 2n\pi, \quad x = \frac{5\pi}{3} + 2n\pi$$ General solution

Find all solutions of $\cos x + 1 = \sin x$ in the interval $[0, 2\pi)$.

Solution

It is not clear how to rewrite this equation in terms of a single trigonometric function. Notice what happens when you square each side of the equation.

Write original equation.

quare each side.

ythagorean identity

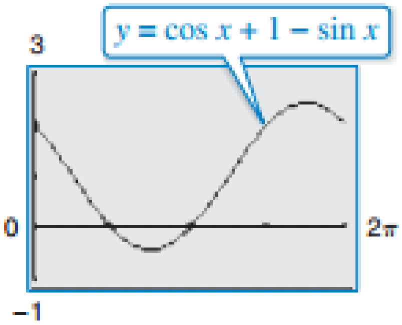
ombine like terms.

actor.

Setting each factor equal to zero produces the following.

2 c

Of the three possible solutions, $x = 3\pi/2$ is extraneous. So, in the interval $[0, 2\pi)$, the only solutions are $x = \pi/2$ and $x = \pi$. The graph of $y = \cos x + 1 - \sin x$, shown in Figure 5.11, confirms this result because the graph has two x-intercepts (at $x = \pi/2$ and $x = \pi$) in the interval $[0, 2\pi)$.



Summarize the major points

How do you solve trigonometric equations written in quadratic form or containing more than one angle?

Solve trig equations by converting the equation to quadratic form and factoring.

Solve trig equations containing sin and cos of the sin ku and cos ku by solving for u and identifying all the values for which this is true over the given interval.

Summarize what you do if it is a multiple angle.

Cheyenne, WY has a latitude of 41°N. At this latitude, the position of the sun at sunrise can be modeled by:

$$D = 31\sin\left(\frac{2\pi}{365t} - 1.4\right)$$

where t = 1 represents Jan. 1 and D represents the degrees N or S of due east that the sun rises. Use a graphing utility to determine the days that the sun is more than 20° N at sunrise.

Example 12 Surface Area of a Honeycomb

The surface area of a honeycomb is given by the equation

$$S = 6hs + \frac{3}{2}s^2\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right), \qquad 0 < \theta \leq 90°$$

where $h = 2.4$ inches, $s = 0.75$ inch, and θ is the angle indicated in Figure 5.16.

a. What value of θ gives a surface area of 12 square inches?

b. What value of θ gives the minimum surface area?

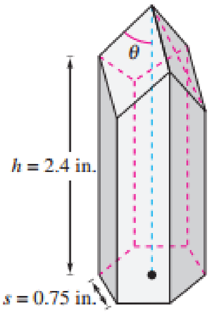


Figure 5.16

Solution

a. Let $h = 2.4$, $s = 0.75$, and $S = 12$.

$$S = 6hs + \frac{3}{2}s^2\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right)$$

$$12 = 6(2.4)(0.75) + \frac{3}{2}(0.75)^2\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right)$$

$$12 = 10.8 + 0.84375\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right)$$

$$0 = 0.84375\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right) - 1.2$$

Using a graphing utility set in *degree* mode, you can graph the function

$$y = 0.84375\left(\frac{\sqrt{3} - \cos x}{\sin x}\right) - 1.2.$$

Using the *zero* or *root* feature, you can determine that

$\theta \approx 49.9°$ and $\theta \approx 59.9°$. See Figure 5.17.

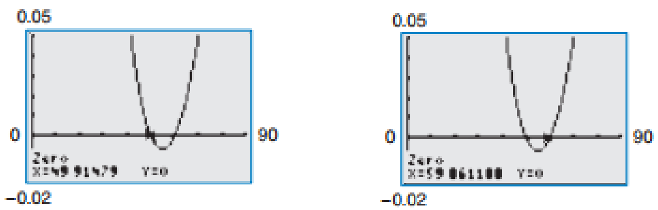


Figure 5.17 $y = 0.84375\left(\frac{\sqrt{3} - \cos x}{\sin x}\right) - 1.2$

b. From part (a), let $h = 2.4$ and $s = 0.75$ to obtain

$$S = 10.8 + 0.84375\left(\frac{\sqrt{3} - \cos\theta}{\sin\theta}\right).$$

Graph this function using a graphing utility set in *degree* mode. Use the *minimum* feature to approximate the minimum point on the graph, which occurs at

$$\theta \approx 54.7°$$

as shown in Figure 5.18. By using calculus, it can be shown that the exact minimum value is

$$\theta = \arccos\left(\frac{1}{\sqrt{3}}\right) \approx 54.7356°.$$