6.1 Law of Sines

Oblique Triangle: triangle that has no right angle

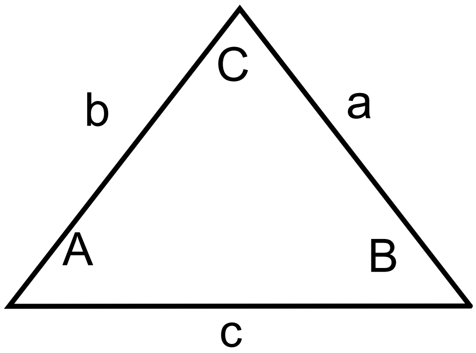
how can we find sides and angles?

To solve an oblique triangle, you need to know the measure of at least one side and the measures of any two other parts of the triangle—two sides, two angles, or one angle and one side. This breaks down into the following four cases.

1. Two angles and any side (AAS or ASA)
2. Two sides and an angle opposite one of them (SSA)
3. Three sides (SSS)
4. Two sides and their included angle (SAS)

Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

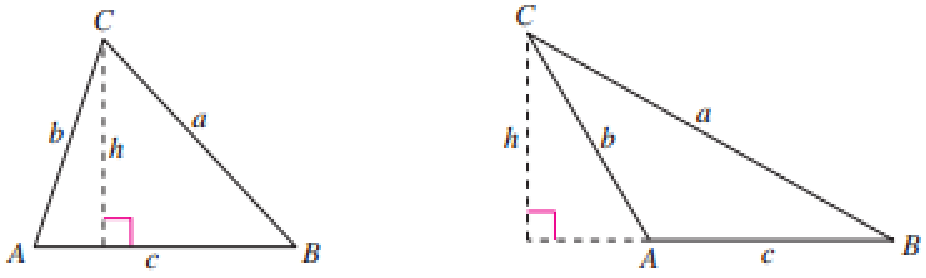


Law of Sines (See the proof on page 464.)

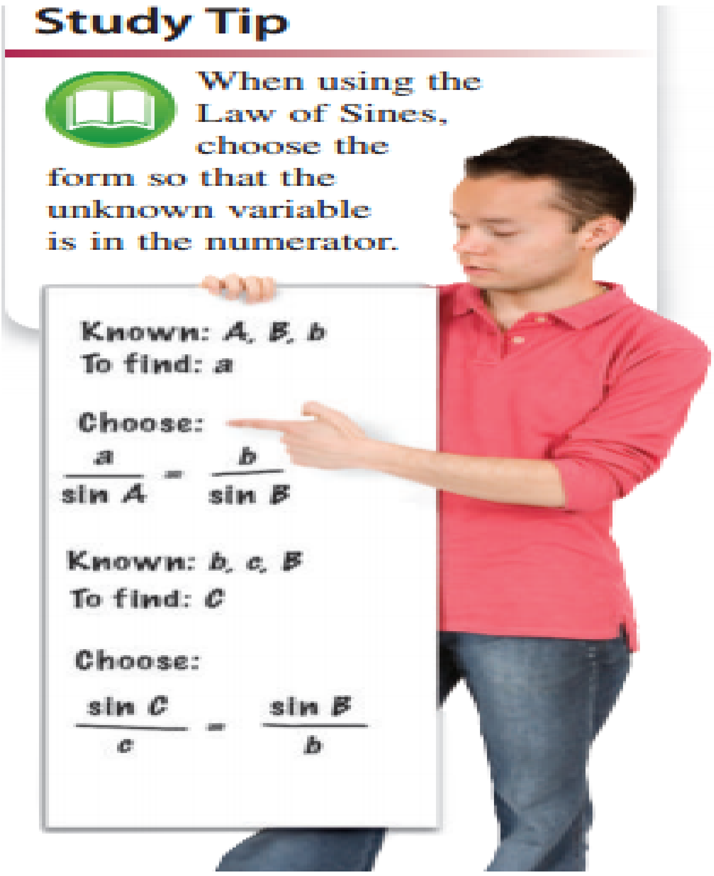
If ABC is a triangle with sides a, b, and c, then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

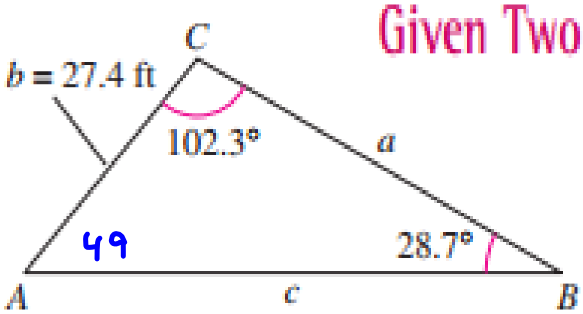
Oblique Triangles



A is acute. *A is obtuse.*


Study Tip
When using the Law of Sines, choose the form so that the unknown variable is in the numerator.
Known: A, B, b
To find: a
Choose:
$\frac{a}{\sin A} = \frac{b}{\sin B}$
Known: b, c, B
To find: C
Choose:
$\frac{\sin C}{c} = \frac{\sin B}{b}$

Given Two Angles and One Side–AAS



$$\frac{a}{\sin 49} = \frac{27.4}{\sin 28.7}$$

$$a = \frac{27.4 \sin 49}{\sin 28.7} = 43.1$$

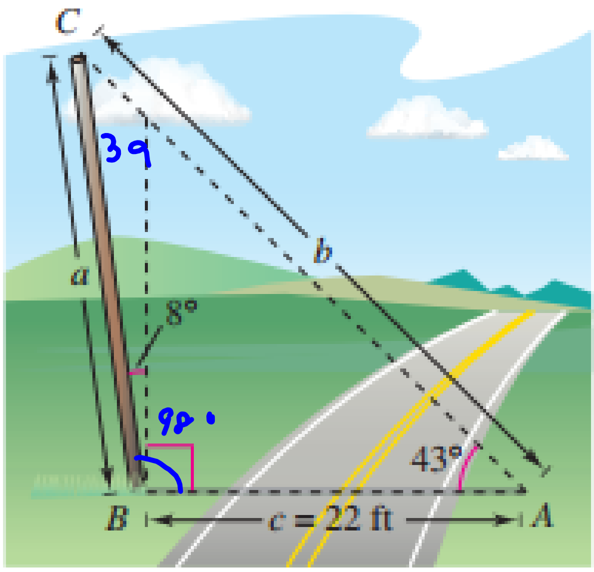
$$\frac{c}{\sin 102.3} = \frac{27.4}{\sin 28.7}$$

$$c = \frac{27.4 \sin 102.3}{\sin 28.7}$$

$c = 55.747$

$c = 55.7$

Given Two Angles and One Side–ASA



$$\frac{22}{\sin 39} = \frac{b}{\sin 98} \qquad b = 34.6 \text{ ft}$$

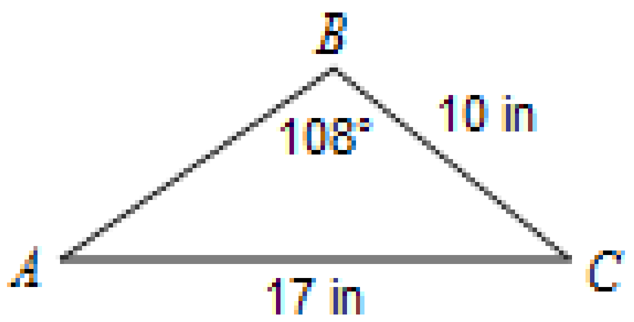
$$\frac{22}{\sin 39} = \frac{a}{\sin 43}$$

$$a \approx 23.8 \text{ ft}$$

Solve the triangle. $A = 31.6^{\circ}$, $C = 42.9^{\circ}$, $a = 10.4$m

Find each measurement indicated. Round your answers to the nearest tenth.

1) Find $m\angle A$



$$\frac{\sin A}{10} = \frac{\sin 108}{17}$$

$$\sin A = \frac{10 \sin 108}{17}$$

$\sin A = 0.559$

$\angle A = 34°$

Find all the missing parts of the triangle.

$m\angle C = 116°, m\angle A = 36°, c = 23$

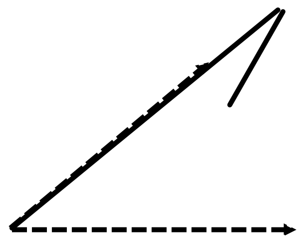
<B =

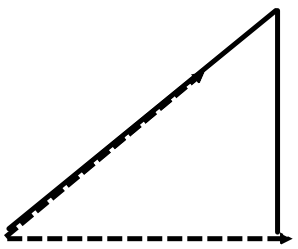
a =

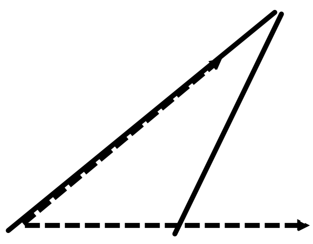
b =

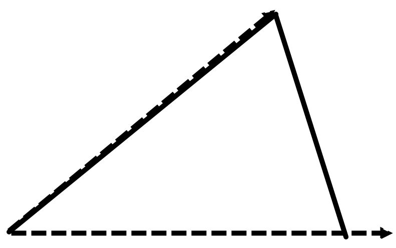
The Ambiguous Case (SSA)

how many possible solutions are there?





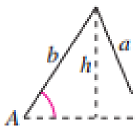
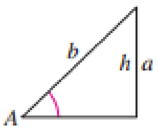
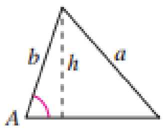
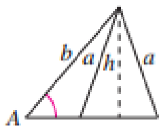
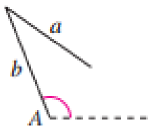
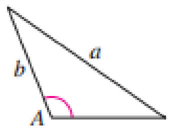




The Ambiguous Case (SSA)

The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. (Notice that $h = b \sin A$.)

	A is acute.	A is acute.	A is acute.	A is acute.	A is obtuse.	A is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a \geq b$	$h < a < b$	$a \leq b$	$a > b$
Possible triangles	None	One	One	Two	None	One

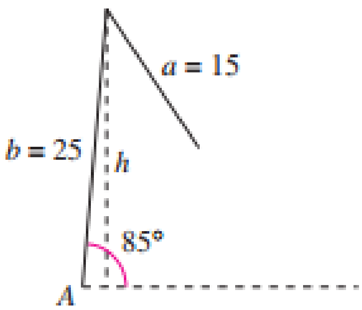
The Ambiguous Case (SSA)

3 possible solutions: 0, 1, or 2 triangles

Steps:

1. Assume there are 2 triangles
2. Solve. If sin $A > 1$: no solution
   (calculator will give error message)
3. Solve for A_1.
4. Solve for A_2. $(180 - A_1)$
5. Solve for third angle.
   If it doesn't exist, there is only one triangle.
   Otherwise, there are 2.

No-Solution Case—SSA

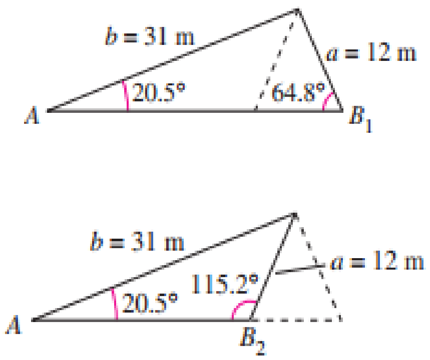


a=15 b = 25 A= 85°

$$\frac{\sin B}{25} = \frac{\sin 85}{15}$$

Two-Solution Case—SSA

$A = 20.5^o$ a= 12 m b=31m



$\frac{\sin B}{31} = \frac{\sin 20.5}{12}$

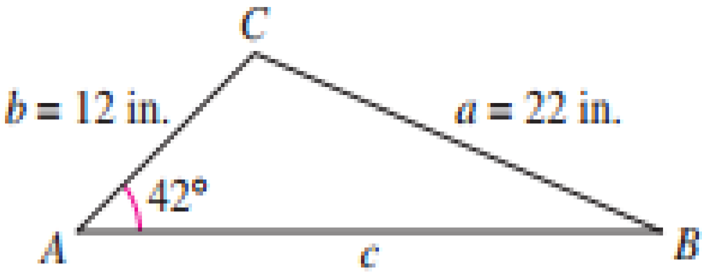
B = 64.8°

A = 20.5°
B = 64.8°
C = 94.7°
a = 12
b = 31
c = 34.2

A = 20.5°
B = 115.2°
C = 44.3°
a = 12
b = 31
c = 23.9

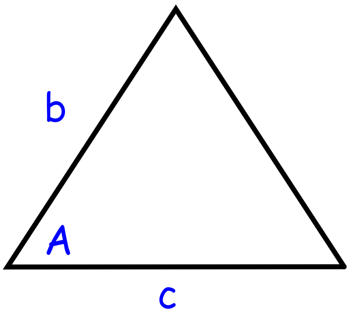
$\frac{c}{\sin 94.7} = \frac{12}{\sin 20.5}$

$\frac{c}{\sin 44.3} = \frac{12}{\sin 20.5}$


C
b = 12 in.
a = 22 in.
42°
A
c
B

Solve the triangle. $A = 25^{\circ}$, $a = 26'$, $b = 54'$

Solve the triangle. $A = 110^{\circ}$, $a = 25'$, $c = 16'$



Area of an Oblique Triangle (SAS only)

Draw in the height of the triangle.
Use SOH-CAH-TOA and the A = bh/2
to come up with a new formula for
the area of a triangle

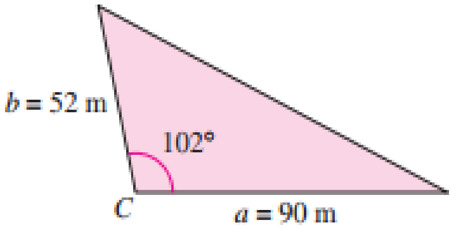
$$\text{Area} = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}(c)(b \sin A)$$
$$= \frac{1}{2}bc \sin A.$$

Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,

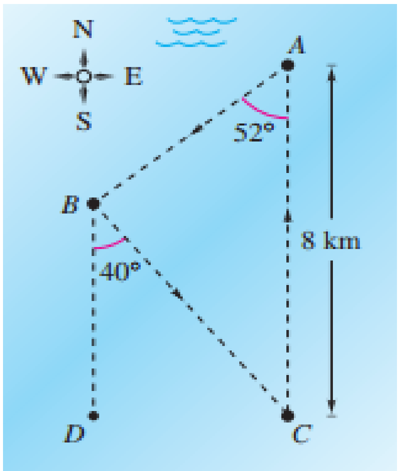
$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B.$$

Find the area of a triangle having side lengths 30' and 48' and an included angle of 40°.



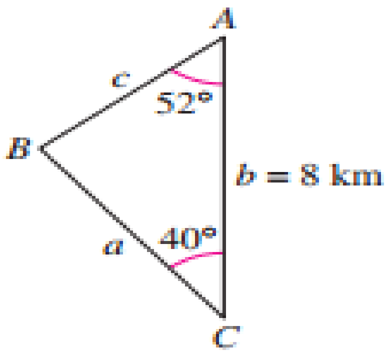
Find the area.

The course for a boat race starts at point A and proceeds in the direction S 52° W to point B, then in the direction S 40° E to point C, and finally back to point A, as shown in Figure 6.10. Point C lies 8 kilometers directly south of point A. Approximate the total distance of the race course.



Because lines BD and AC are parallel, it follows that

$\angle BCA \cong \angle DBC.$



With a partner, answer all the following. Show all your work. One paper per group.

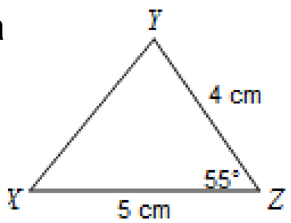
Find the missing parts.

1. <A= 85 <B = 54.2 c = 29in

2. <A= 155 c = 17 a = 17

3. A = 110°, a = 25', c = 16'
4. A = 25°, a = 26', b = 54'

5. Find the area



The course for a bike race starts a point A and proceeds in the direction S 52° W to point B, then in the direction S 40° E to point C, and finally back to A. Point C lies 8 mi directly south of A. How long is the race?