http://www.youtube.com/watch?v=fVq4_HhBK8Y

Airplane

u = unit vector = $\dfrac{v}{\|v\|} = \left(\dfrac{1}{\|v\|}\right)v$

$\langle 6, 8 \rangle$

$$\|v\| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$\sqrt{6^2 + 8^2} = \sqrt{100} = 10$

$\dfrac{\langle 6, 8 \rangle}{10}$

$\dfrac{6}{10}, \dfrac{8}{10}$

$\langle 3/5, 4/5 \rangle$

Find a unit vector in the direction of $\mathbf{v} = \langle -8, 6 \rangle$.

$$\frac{v}{\|v\|}$$

Find a unit vector u in the direction of $v = \langle 7, -3 \rangle$ and verify that the result has magnitude 1.

find the unit vector
then use in

$$\|v\| = \sqrt{(v_1)^2 + (v_2)^2}$$

$$\left\langle \frac{7}{\sqrt{58}}, \frac{-3}{\sqrt{58}} \right\rangle$$

$$\sqrt{\left(\frac{7}{\sqrt{58}}\right)^2 + \left(\frac{-3}{\sqrt{58}}\right)^2} = 1$$

Find the vector v with the given magnitude and same direction as u.

$\|v\| = 3 \qquad u = \langle 4, -4 \rangle$

find the unit vector
multiply it by the magnitude

$u = \sqrt{4^2 + (-4)^2} = \sqrt{32}$

$3\left\langle \frac{4}{\sqrt{32}}, \frac{-4}{\sqrt{32}} \right\rangle$

$\left\langle \frac{12}{\sqrt{32}}, \frac{-12}{\sqrt{32}} \right\rangle$

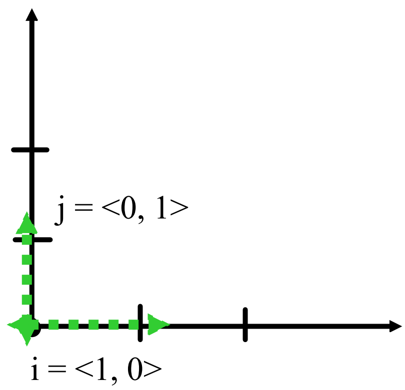
32
16 2
4 4

$\frac{12}{4\sqrt{2}}, \frac{-12}{4\sqrt{2}}$

$\left\langle \frac{3}{\sqrt{2}}, \frac{-3}{\sqrt{2}} \right\rangle$

Standard Unit Vectors:

i = <1, 0>   j = <0, 1>



Linear Combination

Vector $\mathbf{v} = \langle v_1, v_2 \rangle$

can also be represented as:

$$\mathbf{v} = \langle v_1, v_2 \rangle$$
$$= v_1\langle 1, 0\rangle + v_2\langle 0, 1\rangle$$
$$= v_1\mathbf{i} + v_2\mathbf{j}$$

⟨6, 8⟩ ⇒ 6i + 8j

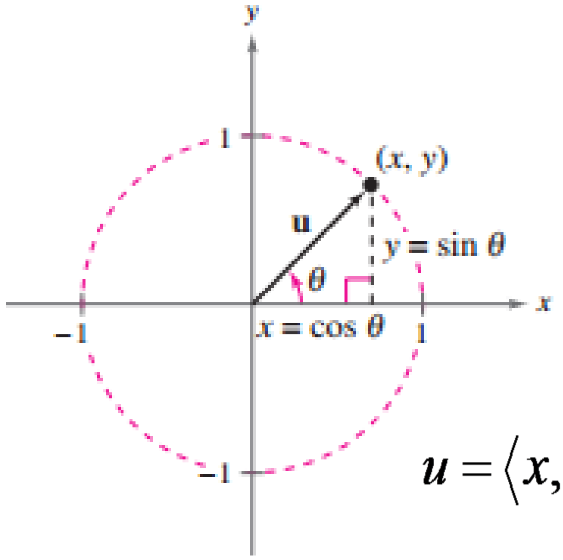
Let u = -3i+8j and v = 2i - j.
Find 2u - 3v

component form $\langle -12, 19 \rangle$

linear combination form $-12i + 19j$

Let **v** = $\langle -5, 3\rangle$. Write **v** as a linear combination of the standard unit vectors **i** and **j**.

Direction Angles



If **u** is a unit vector such that θ is the angle (measured counter clockwise) from the positive x-axis to u, then the terminal point of **u** lies on the unit circle

$$u = \langle x, y \rangle = \langle \cos\theta, \sin\theta \rangle = \cos\theta i + \sin\theta j$$

The angle θ is the direction angle of vector u.

$$v = \|v\|\langle \cos\theta, \sin\theta \rangle = \|v\|(\cos\theta)i + \|v\|(\sin\theta)j$$

Vector **v** has direction angle $\theta = 30^o$ and magnitude 6. Find **v**.

$$v = \|6\|\left\langle \cos 30^o, \sin 30^o \right\rangle = \|v\|(\cos\theta)i + \|v\|(\sin\theta)j$$

v=ai + bj

$$\tan\theta = \frac{\|v\|\sin\theta}{\|v\|\cos\theta} = \frac{b}{a}$$

You must pay attention to what quadrant you are in!

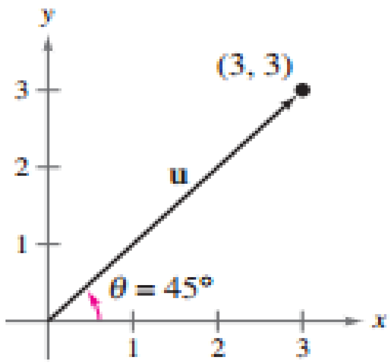
Find the direction angle of each vector.

a. u = 3i + 3j

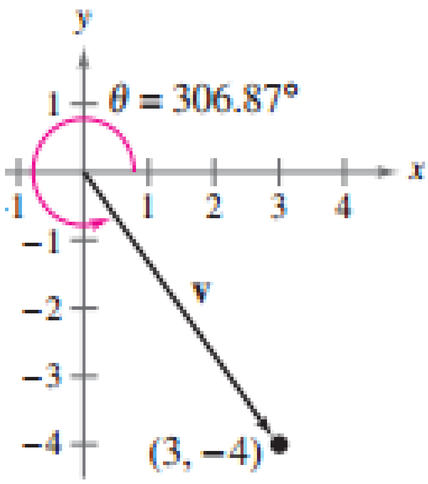
$$\tan\theta = \frac{\|v\|\sin\theta}{\|v\|\cos\theta} = \frac{b}{a}$$

$\frac{3}{3} = 1$

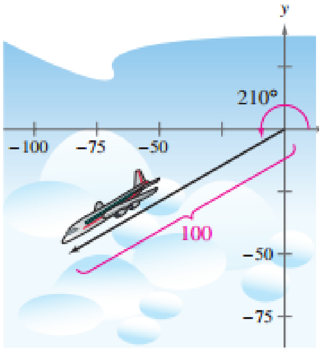
u=ai+bj



b. v = 3i - 4j



Find the component form of the vector that represents the velocity of an airplane descending at a speed of 100 miles per hour at an angle of 30° below the horizontal, as shown in Figure 6.32.



The velocity vector **v** has a magnitude of 100 and a direction angle of $\theta = 210°$

$$\mathbf{v} = \|\mathbf{v}\|(\cos\theta)\mathbf{i} + \|\mathbf{v}\|(\sin\theta)\mathbf{j}$$
$$= 100(\cos 210°)\mathbf{i} + 100(\sin 210°)\mathbf{j}$$
$$= 100\left(-\frac{\sqrt{3}}{2}\right)\mathbf{i} + 100\left(-\frac{1}{2}\right)\mathbf{j}$$
$$= -50\sqrt{3}\,\mathbf{i} - 50\mathbf{j}$$
$$= \langle -50\sqrt{3}, -50\rangle$$

To find the angle:

$$\tan\theta = \frac{-50}{-50\sqrt{3}}$$

You can check that **v** has a magnitude of 100 as follows.

$$\|\mathbf{v}\| = \sqrt{(-50\sqrt{3})^2 + (-50)^2}$$
$$= \sqrt{7500 + 2500}$$
$$= \sqrt{10{,}000}$$
$$= 100$$

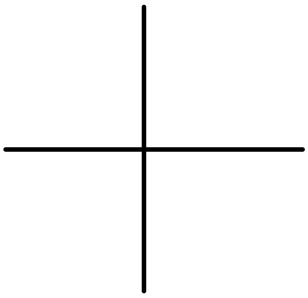
Solution checks. ✓

Using Vectors to Find Speed and Direction *Example 10 p.416*

An airplane is traveling at a speed of 500 mph with a bearing of 330^{o}. The airplane encounters a wind blowing 70 mph in the direction N 45^{o} E. What are the resultant speed and direction of the airplane? Bearing starts at N.

What is the magnitude?

Plane:



Wind:

Plane + Wind

magnitude: $\|v\| =$

$\|v\| =$

direction:

$\tan\theta =$

$\theta =$

The initial point of a vector is (-6, 4) and the terminal point is (0, 1)
Write a linear combination of the standard vector.

Find the component form of V given its magnitude and angle.

$||v|| = 4, \ \theta = 45^o$

Applications of Vectors

force / velocity / tension = magnitude / length

also think: Geometry, right triangle trig, law of cosines and sines

Highlight of Important Vocab / Formulas:

Component Form: **v = < $\mathbf{v_1}$, $\mathbf{v_2}$ >**
found by subtracting x values and y values of points

Magnitude: $||v||$
found by: $\sqrt{v_1^2 + v_2^2}$ *or the distance formula*

Unit Vector: $\dfrac{\mathbf{v}}{||\mathbf{v}||}$

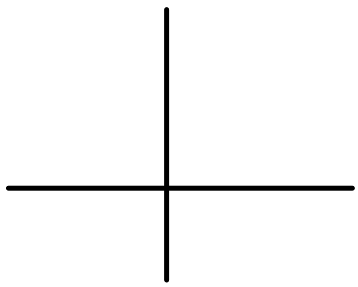
Linear Combination of v: $\mathbf{v_1 i + v_2 j}$

Direction Angles: $\mathbf{v} = ||\mathbf{v}|| \cos\theta\ \mathbf{i} + ||\mathbf{v}|| \sin\theta\ \mathbf{j}$

If v = ai + bj, then $\tan\theta = \dfrac{b}{a}$

Using Vectors to Find Speed and Direction

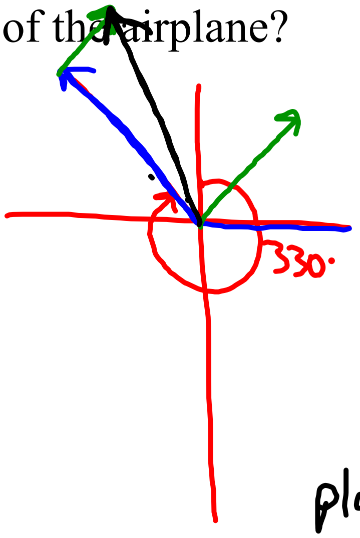
An airplane is traveling at a speed of 500 mph with a bearing of 330^o. The airplane encounters a wind blowing 70 mph in the direction N 45^o E. What are the resultant speed and direction of the airplane?



Using Vectors to Find Speed and Direction *Example 10 p.416*

° clockwise from N $||v|| = 500$

An airplane is traveling at a speed of 500 mph with a bearing of 330°. The airplane encounters a wind blowing 70 mph in the direction N 45° E. What are the resultant speed and direction of the airplane?



plane:

$500\cos 120, \ 500\sin 120$

$500\left(-\frac{1}{2}\right), \ 500\frac{\sqrt{3}}{2}$

$\langle -250, 250\sqrt{3} \rangle$

wind: $\langle 35\sqrt{2}, 35\sqrt{2} \rangle$

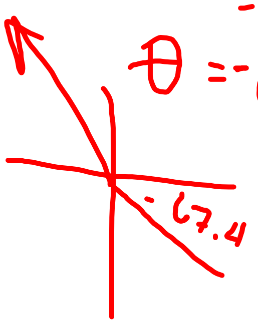
plane wind:

$\langle -200.5, 482.5 \rangle$

Speed $= ||v|| = \sqrt{(-200.5)^2 + (482.5)^2}$

$||v|| = 522.5$ mph

$\tan\theta = \dfrac{482.5}{-200.5}$



$\theta = -67.4 + 180 = \boxed{112.6°}$