

Perform the indicated operation.

$$1) \begin{aligned} f(x) &= 2x - 1 \\ g(x) &= x^2 + 3 \\ \text{Find } f(g(x)) \end{aligned}$$

$$2x^2 + 5$$

$$3) \begin{aligned} f(x) &= x^2 + 4 \\ g(x) &= 4x + 5 \\ \text{Find } f(g(x)) \end{aligned}$$

$$16x^2 + 40x + 29$$

$$\begin{aligned} &(4x+5)^2 + 4 \\ &(4x+5)(4x+5) \\ &16x^2 + 40x + 25 + 4 \end{aligned}$$

$$2) \begin{aligned} g(n) &= n + 2 \\ h(n) &= n^3 + 2n \\ \text{Find } g(h(n)) \end{aligned}$$

$$n^3 + 2n + 2$$

$$4) \begin{aligned} f(t) &= -3t + 3 \\ g(t) &= t^2 - 4 \\ \text{Find } f(g(t)) \end{aligned}$$

$$-3t^2 + 15$$

# 6.4

## Notes: Inverse Functions

# Notes! Inverse Functions

Original Function

Inverse Function

$$f(x)$$

$$g(x) =$$

$$f^{-1}(x)$$

$$g^{-1}(x)$$

# Notes! Inverse Functions

## Four Representations: Tables--

### Original Function

x	f(x)
5	6
3	8
1	10
-1	12
-3	14
-5	16
-7	18

### Inverse Function

x	$f^{-1}(x)$
6	5
8	3
10	1
12	-1
14	-3
16	-5
18	-7

Switch the x and y in the table

1.  $f(x) = 3x + 5$

Numerically (table)		Graphically (Graph)	
a. Make a table of values for $f(x)$		In BLACK graph $f(x)$ In BLUE graph $f^{-1}(x)$ In ANOTHER COLOR graph $f(f^{-1}(x))$ LABEL YOUR GRAPHS!	
$x$	-2   -1   0   1   2		
$f(x)$	-1   2   5   8   11		
b. Make a table of values for $f^{-1}(x)$			
$x$	-1   2   5   8   11		
$f^{-1}(x)$	-2   -1   0   1   2		

Analytically (Equation)	Verbally (Words)
<p>a. Find <math>f^{-1}(x)</math> algebraically.</p> $y = 3x + 5$ $x = \frac{y - 5}{3}$ $f^{-1}(y) = \frac{y - 5}{3}$ <p>b. Show that <math>f(f^{-1}(x)) = x</math></p> $3\left(\frac{x-5}{3}\right) + 5 = x$ <p>c. Show that <math>f^{-1}(f(x)) = x</math></p> $\frac{3x + 5 - 5}{3} = \frac{3x}{3} = x$	<p>a. Explain how you know from the graph that <math>f(x)</math> and <math>f^{-1}(x)</math> are inverses.</p> <p>They reflect about the line symmetry of <math>y=x</math>.</p> <p>b. Explain how you know from the algebra that <math>f(x)</math> and <math>f^{-1}(x)</math> are inverses.</p> $f(f^{-1}(x)) = x \quad f^{-1}(f(x)) = x$ <p>c. Explain how you know from the tables that <math>f(x)</math> and <math>f^{-1}(x)</math> are inverses.</p> <p>Switch the <math>x</math> and <math>y</math> in the table</p>

# Notes! Inverse Functions

Four Representations: Equations--

Replace  $f(x)$  with  $y$

Switch the  $x$  and  $y$  and solve for  $y$

Original Function

$$f(x) = 2x + 3$$

$$y = 2x + 3$$

$$x = \frac{2y + 3}{2}$$

$$\frac{x-3}{2} = \frac{2y}{2}$$

$$y = \frac{x-3}{2}$$

Inverse Function

$$f^{-1}(x) = \frac{x-3}{2}$$

# Notes! Inverse Functions

*Try it on your own:*

Original Function

Inverse Function

$$f(x) = 5x + 6$$

$$f^{-1}(x) = \frac{x-6}{5}$$

$$\begin{aligned}y &= 5x + 6 \\x &= \frac{y-6}{5} \\x-6 &= \frac{5y}{5}\end{aligned}$$

# Notes! Inverse Functions

*Try it on your own:*

Original Function

Inverse Function

$$f(x) = \frac{3x - 1}{2}$$

$$f^{-1}(x) = \frac{2x+1}{3} \text{ or } \frac{2}{3}x + \frac{1}{3}$$

$$y = \frac{3x-1}{2}$$

$$2x = \frac{3y-1}{2}$$

$$2x = 3y - 1$$

$$\frac{2x+1}{3} = \frac{3y}{3}$$

$$\frac{2}{3}x + \frac{1}{3}$$



# Notes! Inverse Functions

**Verify** that the two functions are inverses of one another.

Do the composite of your original and your inverse. If both are equal to  $x$  then they are inverses.

$$f(f^{-1}(x)) = x \quad f^{-1}(f(x)) = x$$

$$f(x) = 4x + 2$$

$$4\left(\frac{1}{4}x - \frac{1}{2}\right) + 2$$

$$x - 2 + 2$$

$$x$$

$$f^{-1}(x) = \frac{1}{4}x - \frac{1}{2}$$

$$\frac{1}{4}(4x + 2) - \frac{1}{2}$$

$$x + \frac{1}{2} - \frac{1}{2}$$

$$x$$

## Inverse Functions

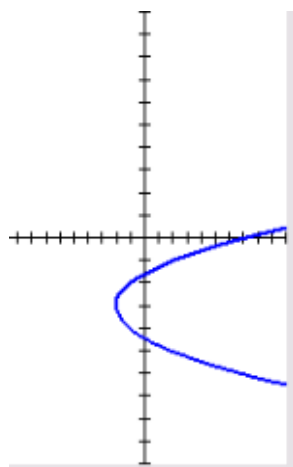
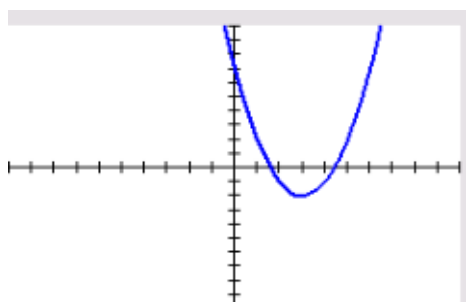
Functions  $f$  and  $g$  are inverses of each other provided:

$$f(g(x)) = x \quad \text{and} \quad g(f(x)) = x$$

The function  $g$  is denoted by  $f^{-1}$ , read as "f inverse".

Given any function, you can always find its inverse relation by switching  $x$  and  $y$ . Then solve for  $y$ .

**Recall:** How can you tell whether a relation is a function?



One-to-One

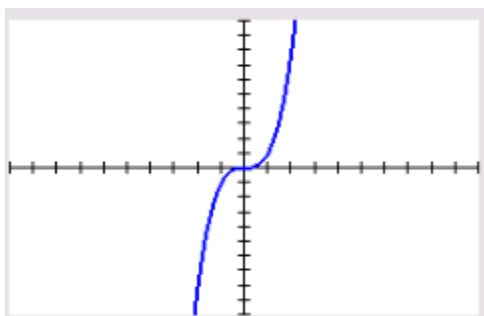
Both the original and the inverse are functions.

### Vertical line test

If a vertical line intersects the graph of a function  $f$  more than once, then the inverse of  $f$  is itself a function.

### Horizontal line test

If a horizontal line intersects the graph of a function  $f$  more than once, then the inverse of  $f$  is itself a function.

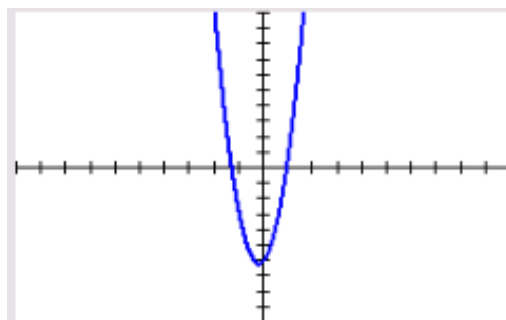
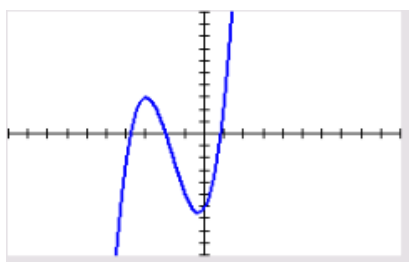
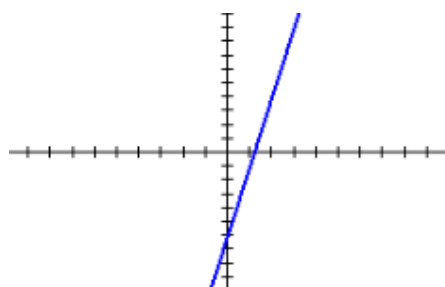
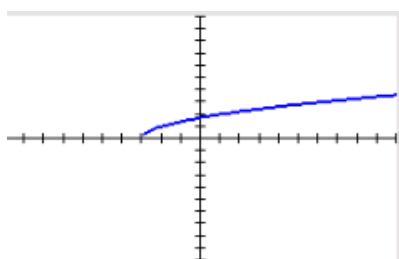


Vertical line Test

Horizontal line test

If both horizontal and vertical line tests hold true, then it is one-to-one.

Is it a one-to-one function?



## Inverse Functions

*Verify* that the two functions are inverses of one another.

Now you try...

$$f(x) = 3x - 5 \qquad f^{-1}(x) = \frac{1}{3}x + \frac{5}{3}$$

# Notes! Graphing Inverse Functions

## Four Representations: Graphing--

### Steps to graphing an inverse function:

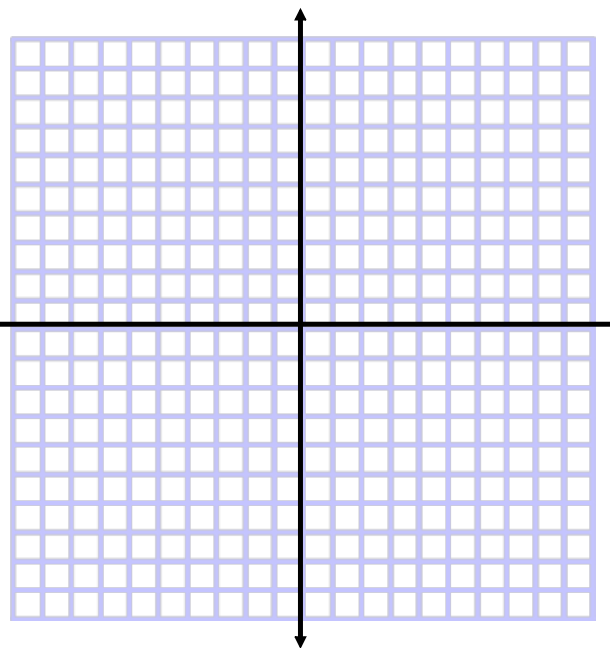
1. Find the inverse function
2. Create a table
3. Evaluate the range
4. Plot inverse points
5. Verify using the  $y=x$  reflection line.



$$y = \frac{1}{4}x - 3$$

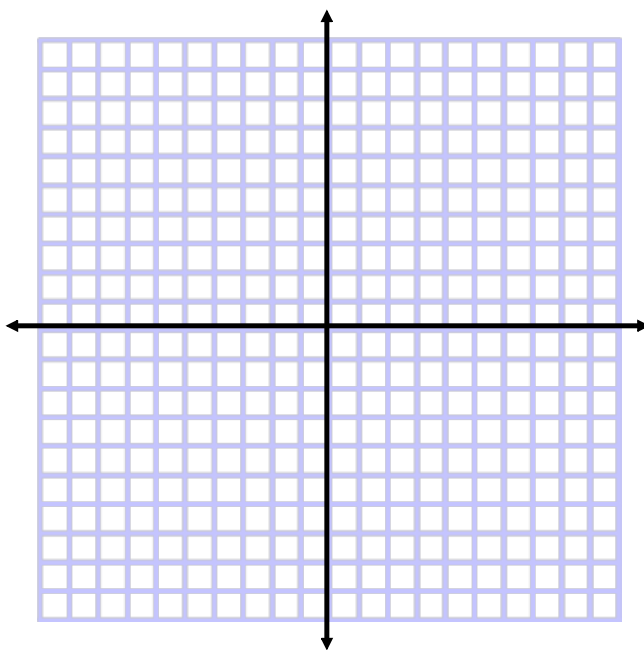
X	Y

X	Y



$$y = \frac{1}{2}x + 4$$

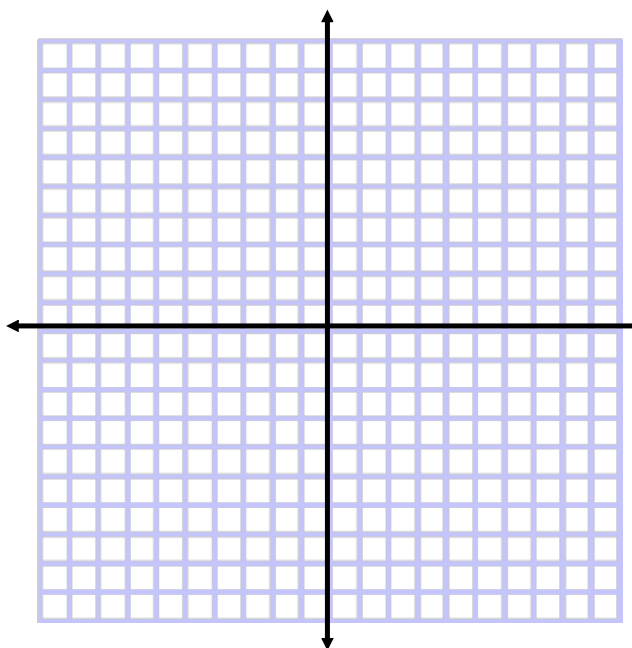
X	Y




What about this...

$$y = (x - 2)^2$$

X	Y



 4 representations (Inverses) WS #5.pdf



## Attachments

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4 representations (Inverses) WS #5.pdf