

## Classroom Overview

Cell phones

Online Grades

HW

Scientific Calculator daily

Handouts

Absent

Grade Breakdown:

Tests: 65%

Assignments: 15%

**PSAT** 5%

Final Exam 15%

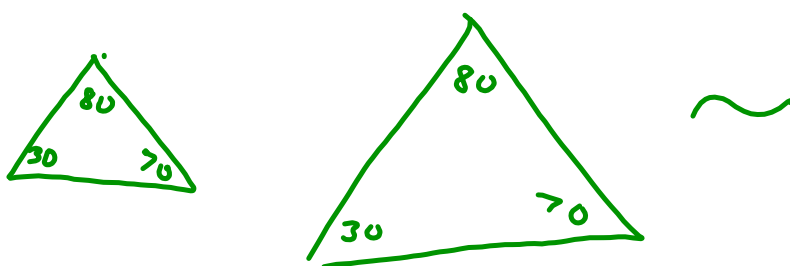
**Learning Goal:** Today I will learn about ratios and proportions.

**Success Criteria:** I am able to set up a ratio and a proportion, and use them to solve for a variable.

# 7.1 Ratios and Proportions

## Proving Similar Triangles

Two triangles are similar if and only if corresponding angles are congruent and corresponding sides are proportional.





### Problem 3 Using an Extended Ratio

The lengths of the sides of a triangle are in the extended ratio 3 : 5 : 6. The perimeter of the triangle is 98 in. What is the length of the longest side?

Sketch the triangle. Use the extended ratio to label the sides with expressions for their lengths.



$$3x + 5x + 6x = 98$$

The perimeter is 98 in.

$$14x = 98$$

Simplify.

$$x = 7$$

Divide each side by 14.

The expression that represents the length of the longest side is  $6x$ .  $6(7) = 42$ , so the length of the longest side is 42 in.



**Got It?** 3. The lengths of the sides of a triangle are in the extended ratio 4 : 7 : 9. The perimeter is 60 cm. What are the lengths of the sides?

$$4x + 7x + 9x = 60$$

$$\frac{20x}{20} = \frac{60}{20}$$

$$x = 3$$

$$12, 21, 27$$

An equation that states that two ratios are equal is called a **proportion**. The first and last numbers in a proportion are the **extremes**. The middle two numbers are the **means**.

$$\begin{array}{c} \downarrow \text{extremes} \downarrow \\ 2 : 3 = 4 : 6 \\ \uparrow \quad \uparrow \\ \text{means} \end{array}$$

$$\begin{array}{l} \text{extremes} \rightarrow \\ \text{means} \rightarrow \end{array} \begin{array}{c} \textcircled{2} = \textcircled{4} \\ \textcircled{3} = \textcircled{6} \end{array}$$

$$\frac{2}{3} = \frac{4}{6}$$

$$12 = 12$$

take note

### Key Concept Cross Products Property

#### Words

In a proportion, the product of the **extremes** equals the product of the **means**.

#### Symbols

If  $\frac{a}{b} = \frac{c}{d}$ , where  $b \neq 0$  and  $d \neq 0$ , then  $ad = bc$ .

#### Example

$$\frac{2}{3} = \frac{4}{6}$$

$$2 \cdot 6 = 3 \cdot 4$$

$$12 = 12$$

## Ratios and Proportions

Solve a proportion using cross multiplication.

$$\frac{a}{b} = \frac{c}{d}$$

$$a \cdot d = b \cdot c$$

### Problem 4 Solving a Proportion

**Algebra** What is the solution of each proportion?

**A**  $\frac{6}{x} = \frac{5}{4}$   
 $6(4) = 5x$   
 $24 = 5x$   
 $x = \frac{24}{5}$

Cross Products Property

Simplify.

Solve for the variable.

The solution is  $\frac{24}{5}$  or 4.8.

**B**  $\frac{y+4}{9} = \frac{y}{3}$   
 $3(y+4) = 9y$   
 $3y + 12 = 9y$   
 $12 = 6y$   
 $y = 2$

The solution is 2.

**Got It?** 4. What is the solution of each proportion?

a.  $\frac{9}{2} = \frac{a}{14}$

$$9(14) = 2(a)$$

$$\frac{126}{2} = \frac{2a}{2}$$

$$a = 63$$

b.  $\frac{15}{m+1} = \frac{3}{m}$

$$15(m) = 3(m+1)$$

$$15m = 3m + 3$$

$$- 3m \quad - 3m$$

$$\frac{12m}{12} = \frac{3}{12}$$

$$m = \frac{1}{4}$$



Solve the proportions

$$\frac{3}{y} = \frac{15}{35}$$

$$\begin{aligned} \frac{15y}{15} &= \frac{105}{15} \\ y &= 7 \end{aligned}$$

$$\frac{3k}{27} = \frac{2}{3}$$

$$\begin{aligned} 9k &= 2(27) \\ \frac{9k}{9} &= \frac{54}{9} \\ k &= 6 \end{aligned}$$

## Ratios and Proportions

Solve a proportion using cross multiplication.

$$\frac{14}{x-1} = \frac{7}{17}$$

$$238 = 7(x-1)$$

$$238 = 7x - 7$$
$$+7 \quad +7$$

$$\frac{245}{7} = \frac{7x}{7}$$
$$x = 35$$

take note

### Key Concept Properties of Proportions

$a$ ,  $b$ ,  $c$ , and  $d$  do not equal zero.

#### Property

(1)  $\frac{a}{b} = \frac{c}{d}$  is equivalent to  $\frac{b}{a} = \frac{d}{c}$ .

(2)  $\frac{a}{b} = \frac{c}{d}$  is equivalent to  $\frac{a}{c} = \frac{b}{d}$ .

(3)  $\frac{a}{b} = \frac{c}{d}$  is equivalent to  $\frac{a+b}{b} = \frac{c+d}{d}$ .

#### How to Apply It

Write the reciprocal of each ratio.

$\left(\frac{2}{3} = \frac{4}{6}\right)$  becomes  $\frac{3}{2} = \frac{6}{4}$ .

Switch the means.

$\frac{2}{3} \swarrow \frac{4}{6}$  becomes  $\frac{2}{4} = \frac{3}{6}$ .

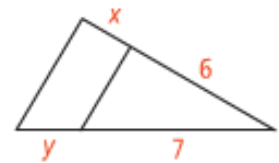
In each ratio, add the denominator to the numerator.

$\frac{2}{3} = \frac{4}{6}$  becomes  $\frac{2+3}{3} = \frac{4+6}{6}$ .



### Problem 5 Writing Equivalent Proportions

In the diagram,  $\frac{x}{6} = \frac{y}{7}$ . What ratio completes the equivalent proportion  $\frac{x}{y} = \frac{\square}{\square}$ ? Justify your answer.



#### Method 1

$$\frac{x}{6} = \frac{y}{7}$$

$$\frac{x}{y} = \frac{6}{7} \quad \text{Property of Proportions (2)}$$

#### Method 2

$$\frac{x}{6} = \frac{y}{7}$$

$$7x = 6y \quad \text{Cross Products Property}$$

$$\frac{7x}{7y} = \frac{6y}{7y} \quad \text{To solve for } \frac{x}{y}, \text{ divide each side by } 7y.$$

$$\frac{x}{y} = \frac{6}{7} \quad \text{Simplify.}$$

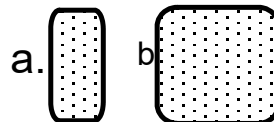
The ratio that completes the proportion is  $\frac{6}{7}$ .



**Got It?** 5. For parts (a) and (b), use the proportion  $\frac{x}{6} = \frac{y}{7}$ . What ratio completes the equivalent proportion? Justify your answer.

a.  $\frac{6}{x} = \frac{\square}{\square}$

b.  $\frac{\square}{\square} = \frac{y+7}{7}$



c. **Reasoning** Explain why  $\frac{6}{x-6} = \frac{7}{y-7}$  is an equivalent proportion to  $\frac{x}{6} = \frac{y}{7}$ .

What does it mean to be in proportion?



## ^Ratios and Proportions

### Ratio

A comparison of two quantities by using division

- can write it a:b, a to b or  $\frac{a}{b}$

- write it in the simplest form (reduce!)

A little boy's height is 3'. His father's height is 6'. Write a ratio of their heights. Boy:Father

$$\frac{3}{6} = \frac{1}{2}$$

$$3:6$$

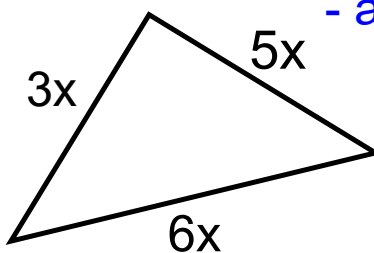
$$1:2$$

$$\frac{3+6}{1+2}$$

### Extended ratio

A ratio that compares three or more numbers

- a:b:c or a:b, b:c and a:c



$$3x+5x+6x=98$$

Proportion - an equation setting 2 ratios equal

## Proportion

An equation that states that two numbers are equal.

$$\frac{2}{3} = \frac{4}{6}$$

**Closure:** Today I learned how to set up ratios and proportions, and solve for an unknown.



$$1) \frac{9}{8} = \frac{4}{x}$$

$\{3.56\}$

$$3) \frac{8}{4} = \frac{2}{n+9}$$

$\{-8\}$

$$2) \frac{5}{4} = \frac{m}{8}$$

$\{10\}$

$$4) \frac{7}{x+5} = \frac{5}{10}$$

$\{9\}$

$$5) \frac{p+5}{8} = \frac{6}{2}$$

{19}

$$7) \frac{r}{r+10} = \frac{10}{6}$$

{-25}

$$6) \frac{6}{2} = \frac{v-8}{10}$$

{38}

$$8) \frac{4}{x} = \frac{3}{x-3}$$

{12}

