9.4 Parametric Equations

Consider the path of an object that is propelled into the air at an angle of 45 degrees. When the initial velocity of the object is 48 feet per second, it can be shown that the object follows the parabolic path.

$$y = -\frac{x^2}{72} + x$$ Rectangular equation

as shown in Figure 9.42. However, this equation does not tell the whole story. Although it does tell you *where* the object has been, it does not tell you *when* the object was at a given point (x, y) on the path. To determine this time, you can introduce a third variable t, called a **parameter.** It is possible to write both x and y as functions of t to obtain the **parametric equations**

$$x = 24\sqrt{2}t$$ Parametric equation for x

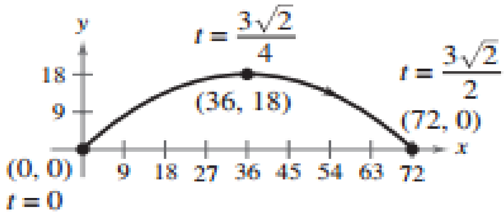
From this set of equations you can determine that at time $t = 0$, the object is at the point $(0, 0)$. Similarly, at time $t = 1$, the object is at the point

$$\left(24\sqrt{2},\ 24\sqrt{2} - 16\right)$$

and so on.

Rectangular equation:
$y = -\frac{x^2}{72} + x$

Parametric equations:
$x = 24\sqrt{2}t$
$y = -16t^2 + 24\sqrt{2}t$



Definition of a Plane Curve

If f and g are continuous functions of t on an interval I, then the set of ordered pairs

$$(f(t), g(t))$$

is a **plane curve** C. The equations given by

$$x = f(t) \qquad \text{and} \qquad y = g(t)$$

are **parametric equations** for C, and t is the **parameter.**

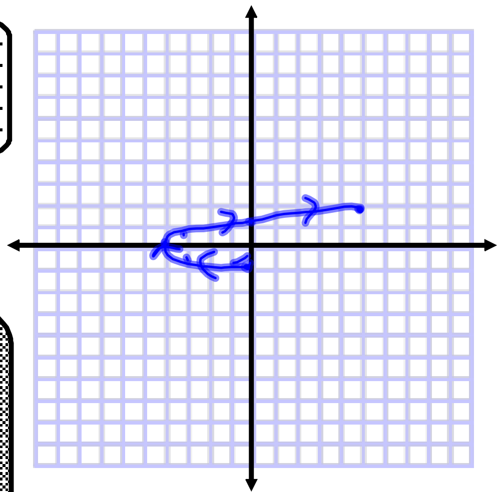
Orientation of the curve: By plotting the resulting points in the order of increasing values of t, you trace the curve in a specific direction.

Sketch the curve given the parametric equations

$x = t^2 - 4 \qquad y = \frac{t}{2} \qquad -2 \leq t \leq 3$

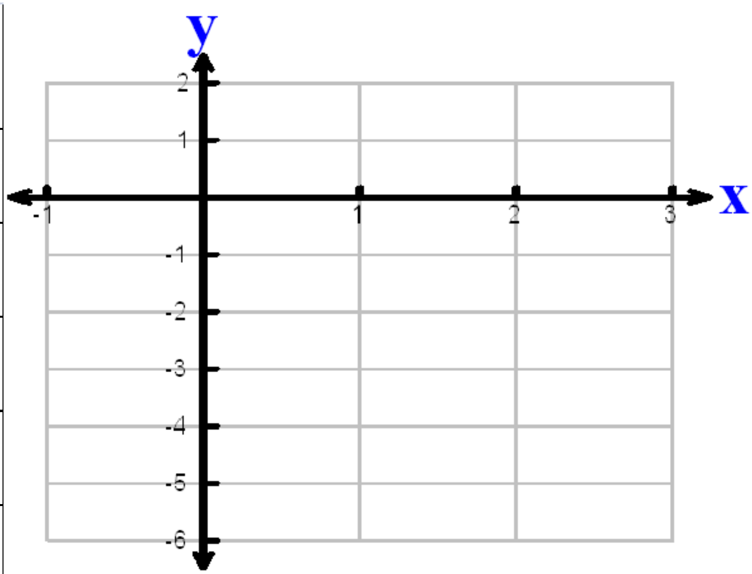
Describe the orientation of the curve

t	-2	-1	0	1	2	3
x	0	-3	-4	-3	0	5
y	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$



Sketch the curve given by the parametric equaons

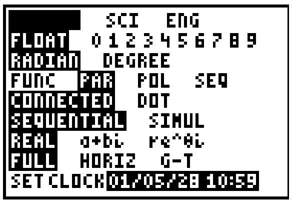
t	$x = t^2 + t$	$y = 2t - 1$
-2		
-1		
-0.5		
0		
1		



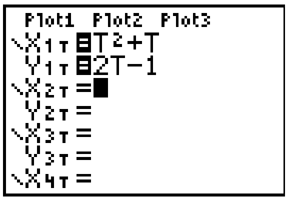
Using a Graphing Ulity in Parametric Mode

Use a graphing ulity to confirm your result.

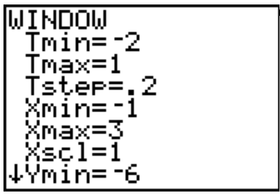
MODE

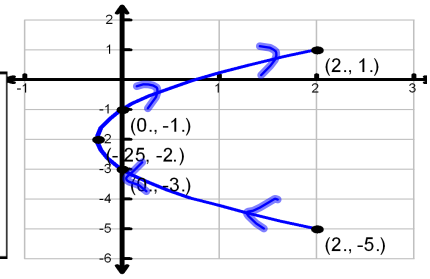


Y=

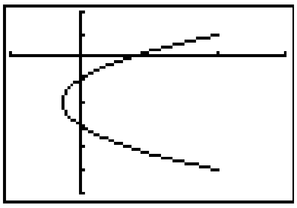


WINDOW

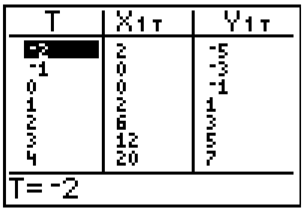




GRAPH



TABLE



t	$x = t^2 + t$	$y = 2t - 1$
-2	2	-5
-1	0	-3
-0.5	-0.25	-2
0	0	-1
1	2	1

Use a graphing calculator set in parametric mode to graph the curve

$x = t$ and $y = 1 - t^2$

Set the viewing window so that $-4 \leq x \leq 4$ and $-12 \leq y \leq 2$. Now, graph the curve with various settings for t. Use the following.

a. $0 \leq t \leq 3$

b. $-3 \leq t \leq 0$

c. $-3 \leq t \leq 3$

Compare the curves given by the different t settings. Repeat this experiment using x = -t.

How does this change the results?

Eliminating the Parameter

Eliminating the parameter is the process of rewriting a parametric equation as a rectangular equation (in terms of x and y)

Steps:

Solve for t in one equaon

Substute in second equaon

When converting equations from parametric to rectangular form, it may be necessary to alter the domain!

Eliminating the Parameter

Many curves that are represented by sets of parametric equations have graphs that can also be represented by rectangular equations (in x and y). The process of finding the rectangular equation is called **eliminating the parameter.**

Parametric equations →	Solve for t in one equation. →	Substitute in second equation. →	Rectangular equation
$x = t^2 - 4$ $y = \frac{1}{2}t$	$t = 2y$	$x = (2y)^2 - 4$	$x = 4y^2 - 4$

Eliminang the Parameter

$$x = t^2 - 4$$
$$y = \frac{1}{2}t$$

Solve for t in one equaon

Substute in second equaon

Rectangular equaon

Graph

Graph

Study Tip

It is important to realize that eliminating the parameter is primarily an aid to curve sketching. When the parametric equations represent the path of a moving object, the graph alone is not sufficient to describe the object's motion. You still need the parametric equations to determine the *position*, *direction*, and *speed* at a given time.

Identify the curve represented by the equations

$$x = \frac{1}{\sqrt{t+1}} \qquad y = \frac{t}{t+1}$$

Solve for t.

Solution

Solving for t in the equation for x produces

[redacted] → [redacted] → [redacted]

Substituting in the equation for y, you obtain the rectangular equation

$$y = \frac{t}{t+1} =$$ [redacted] = [redacted] = [redacted]

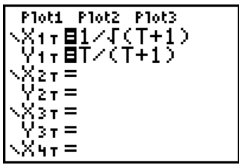
[redacted]

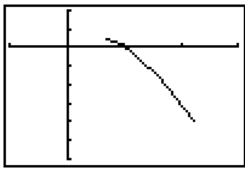
Eliminang the Parameter

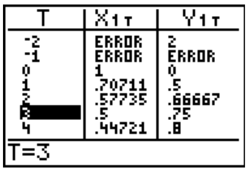
Idenfy the curve represented by the equaons

Parametric Equaons

$$x = \frac{1}{\sqrt{t+1}} \quad and \quad y = \frac{t}{t+1}$$



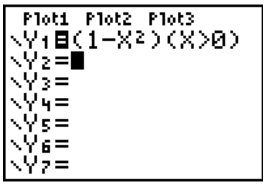


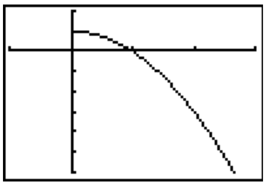


T	X1T	Y1T
-2	ERROR	2
-1	ERROR	ERROR
0	1	0
1	.70711	.5
2	.57735	.66667
3	.5	.75
4	.44721	.8

T=3

- Solve for t in one equaon
- Substute in second equaon
- Rectangular equaon





Domain?

Find a set of parametric equations to represent the graph of $y = 1 - x^2$ using the parameters (a) $t = x$ and (b) $t = 1 - x$.

Solution

a. Letting $t = x$, you obtain the following parametric equations.

$x = t$ — Parametric equation for x

$y = 1 - t^2$ — Parametric equation for y

The graph of these equations is shown in Figure 9.57.

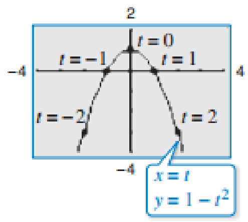


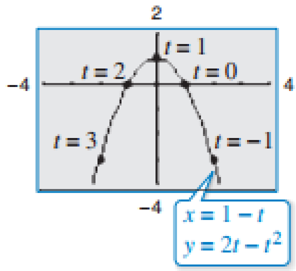
Figure 9.57

b. Letting $t = 1 - x$, you obtain the following parametric equations.

$x = 1 - t$ — Parametric equation for x

$y = 1 - (1 - t)^2 = 2t - t^2$ — Parametric equation for y

The graph of these equations is shown in Figure 9.58. Note that the graphs in Figures 9.57 and 9.58 have opposite orientations.

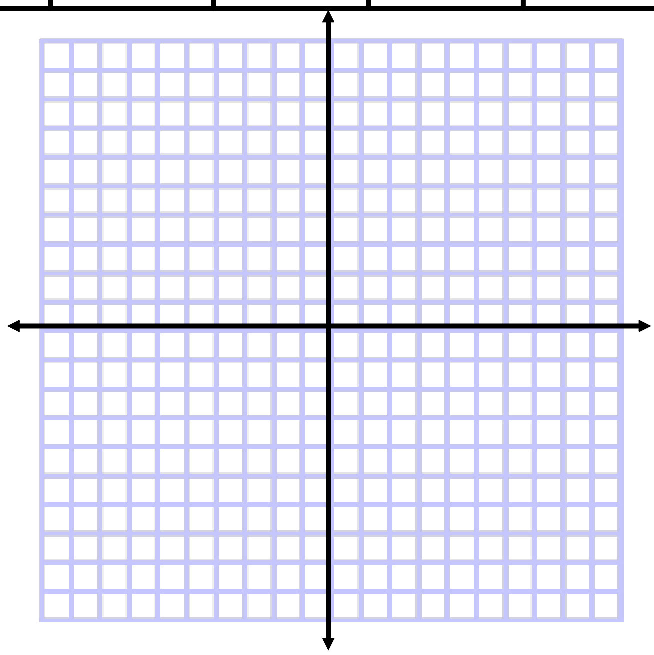


Add to your partner paper

Sketch the curve described by the parametric equations:

$x = t - 3$

$y = t^2 + 1, -1 \leq t \leq 3$



Eliminate the parameter and write the corresponding rectangular equation for:

$x = t - 3$

$y = t^2 + 1$

To eliminate the parameter in equations involving trigonometric functions...

solve for sin and cos

then use trig identities

$$x = 4\cos\theta$$

$$y = 5\sin\theta$$

Rewrite solving for the trig function

$$\cos\theta = \frac{x}{4}$$

$$\sin\theta = \frac{y}{5}$$

Square both sides

$$\cos^2\theta = \frac{x^2}{4^2}$$

$$\sin^2\theta = \frac{y^2}{5^2}$$

Trig Identity

$$\cos^2\theta + \sin^2\theta = 1$$

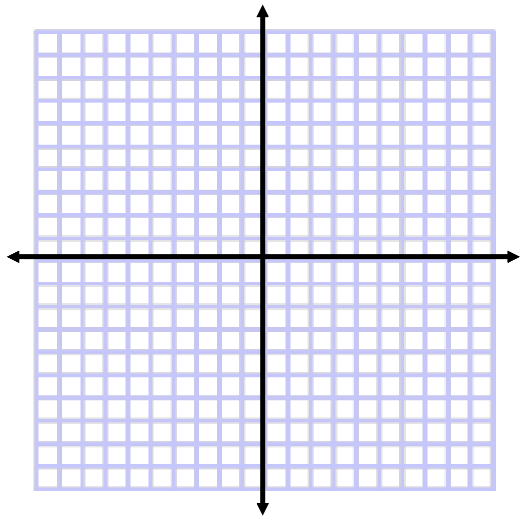
$$\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$$

What type of graph is this?

Eliminate the parameter and graph:

$x = 3\cos\theta$

$y = 4\sin\theta$



Finding Parametric Equations for a Graph

let x = anything in terms of t
substitute that into y to get the second equation

Comparing Plane Curves In Exercises 35 and 36, determine how the plane curves differ from each other.

do a-c

36. (a) $x = 2\sqrt{t}$ (b) $x = 2\sqrt[3]{t}$

$y = 4 - \sqrt{t}$ $y = 4 - \sqrt[3]{t}$

(c) $x = 2(t + 1)$ (d) $x = -2t^2$

$y = 3 - t$ $y = 4 + t^2$

Eliminating the Parameter In Exercises 37–40, eliminate the parameter and obtain the standard form of the rectangular equation.

38. Circle: $x = h + r\cos\theta$, $y = k + r\sin\theta$

Finding Parametric Equations for a Given Graph In Exercises 41–44, use the results of Exercises 37–40 to find a set of parametric equations for the line or conic.

42. Circle: center: $(3, -2)$; radius: 4

Find 2 different sets of parametric equations for the given rectangular equation.
$y = 1 - x^2$

**write your own, do not copy from BOB!*