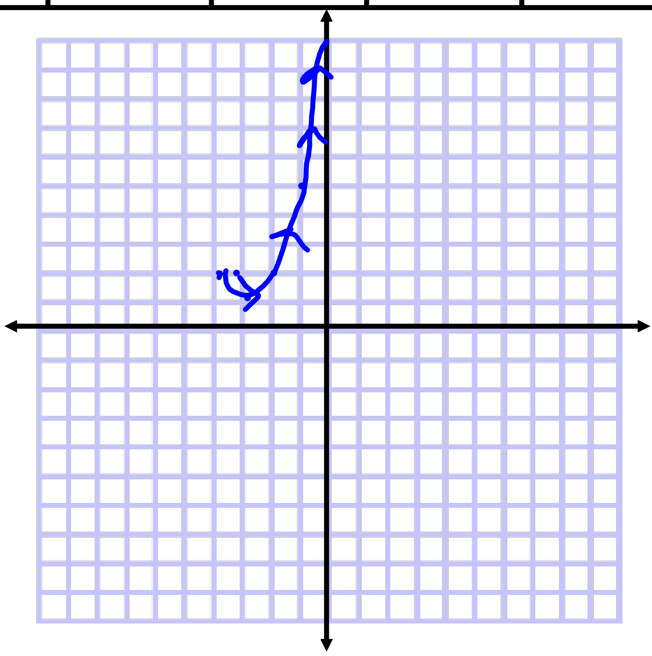
Sketch the curve described by the parametric equations:

$x = t - 3$

$y = t^2 + 1, \ -1 \leq t \leq 3$

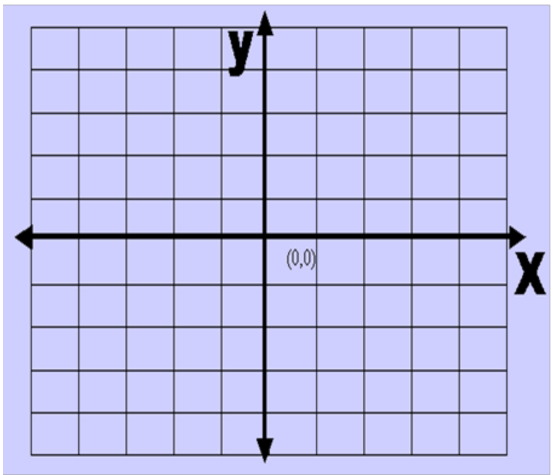
| | | | | | |
|---|---|---|---|---|---|
| t | -1 | 0 | 1 | 2 | 3 |
| x | -4 | -3 | -2 | -1 | 0 |
| y | 2 | 1 | 2 | 5 | 10 |



Polar Coordinates

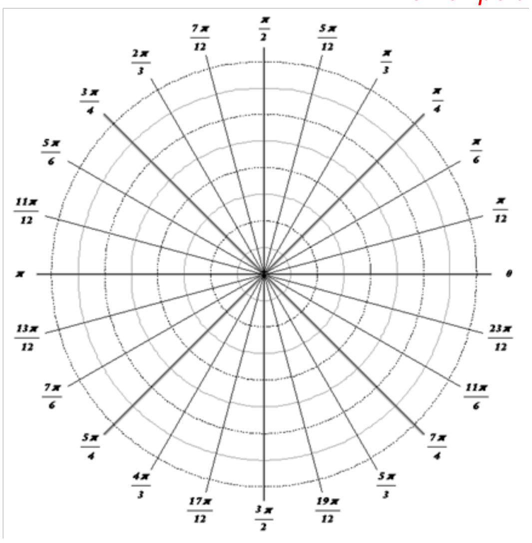
$$(r,\ \theta)$$

You are familiar with plotting with a rectangular coordinate system.



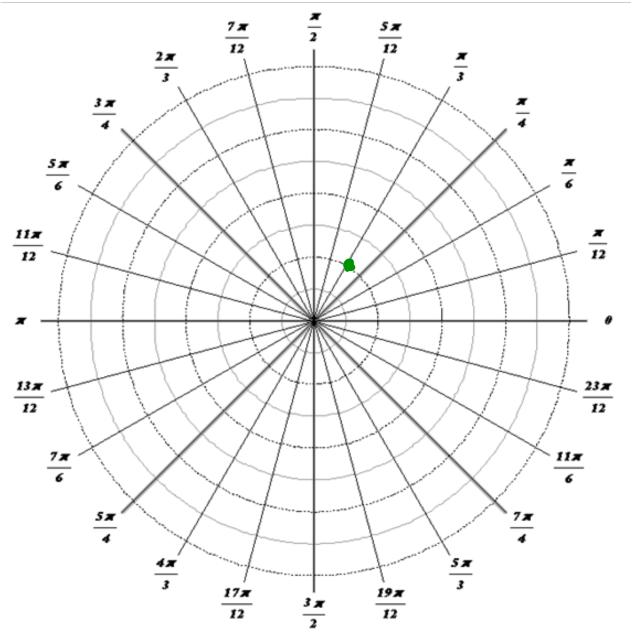

We are going to look at a new coordinate system called the polar coordinate system.

*10° polar graph vs 15° polar graph*



The center of the graph is called the pole.

Angles are measured from the positive *x* axis.




Points are represented by a radius and an angle

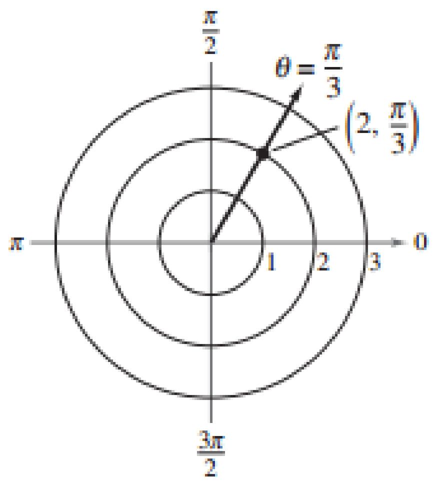
$$(r, \theta)$$

Graph:

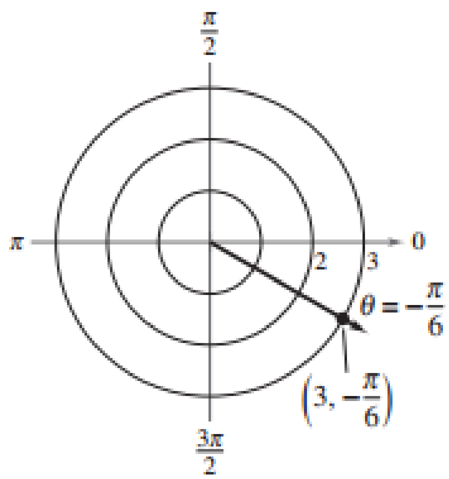
$$\left(2, \frac{\pi}{3}\right)$$

$$\left(2, -\frac{5\pi}{3}\right)$$

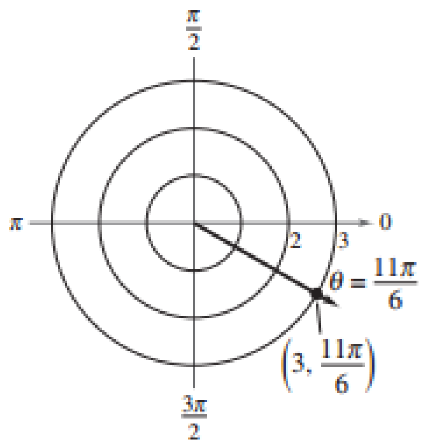
$$\left(-2, \frac{4\pi}{3}\right)$$



$\frac{\pi}{2}$
$\theta = \frac{\pi}{3}$
$\left(2, \frac{\pi}{3}\right)$
$\pi$
1
2
3
0
$\frac{3\pi}{2}$

$\frac{\pi}{2}$
$\pi$
2
3
0
$\theta = -\frac{\pi}{6}$
$\left(3, -\frac{\pi}{6}\right)$
$\frac{3\pi}{2}$

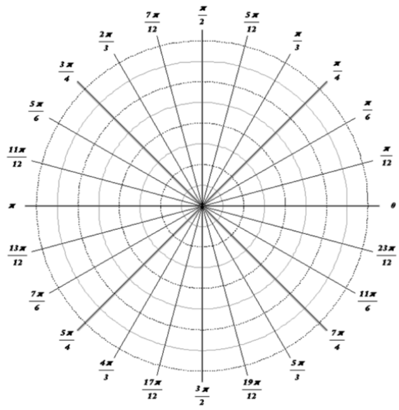
$\frac{\pi}{2}$
$\pi$
2
3
0
$\theta = \frac{11\pi}{6}$
$\left(3, \frac{11\pi}{6}\right)$
$\frac{3\pi}{2}$

Plot the point

$$\left(3, -\frac{3\pi}{4}\right)$$

and find three additional polar representations of this point, using

$$-2\pi < \theta < 2\pi.$$

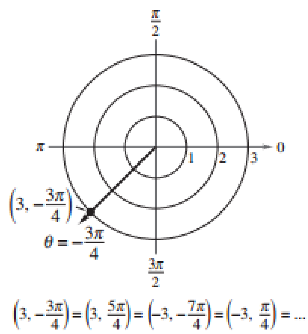


The point is shown in Figure 9.63. Three other representations are as follows.

$$\left(3, -\frac{3\pi}{4} + 2\pi\right) = \left(3, \frac{5\pi}{4}\right) \qquad \text{Add } 2\pi \text{ to } \theta.$$

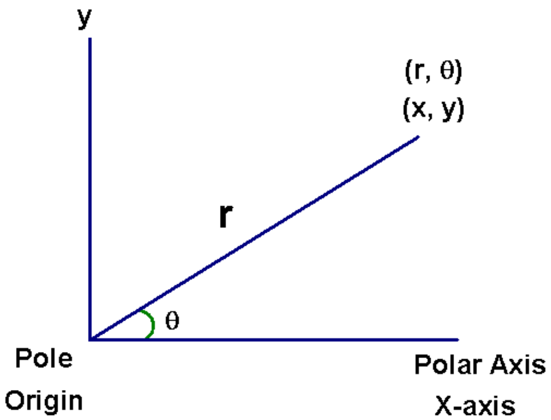
$$\left(-3, -\frac{3\pi}{4} - \pi\right) = \left(-3, -\frac{7\pi}{4}\right) \qquad \text{Replace } r \text{ by } -r\text{; subtract } \pi \text{ from } \theta.$$

$$\left(-3, -\frac{3\pi}{4} + \pi\right) = \left(-3, \frac{\pi}{4}\right) \qquad \text{Replace } r \text{ by } -r\text{; add } \pi \text{ to } \theta.$$



# Coordinate Conversion

Polar to Rectangular




$x = r\cos(\theta)$ $\quad 2\cos\frac{\pi}{3}$

$y = r\sin(\theta)$ $\quad 2\sin\frac{\pi}{3}$

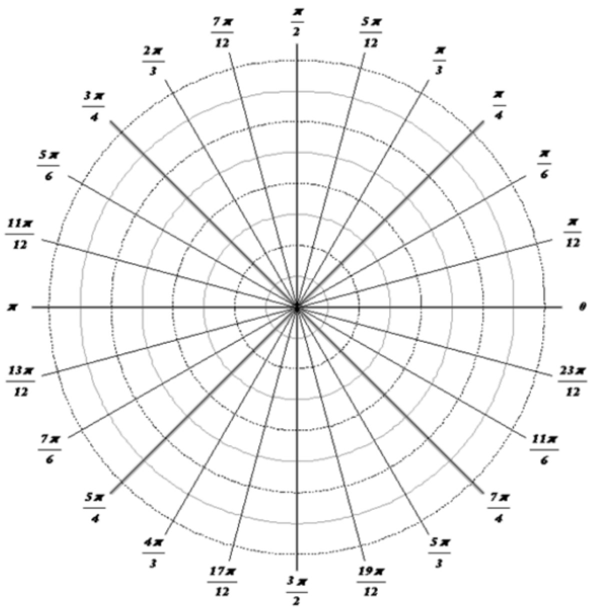
Convert to rectangular coordinates:

$(2, \frac{\pi}{3})$

$r = 2$

$\theta = \frac{\pi}{3}$

$(1, \sqrt{3})$

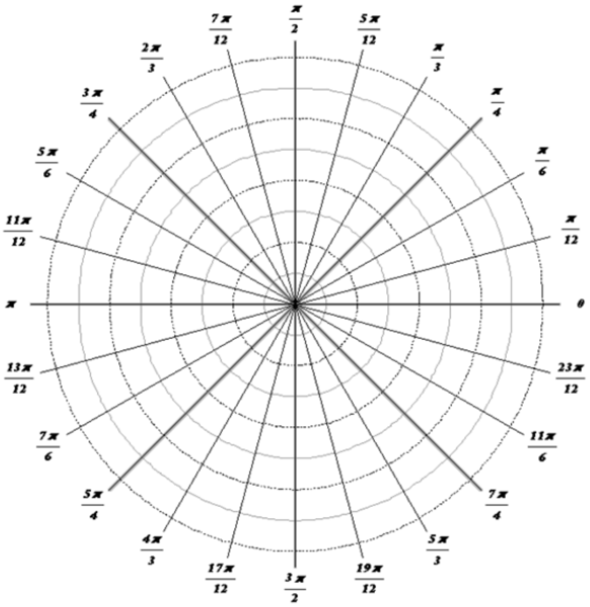



Graph. Then find rectangular coordinates.

$(5, \frac{-\pi}{6})$

$(-5, \frac{\pi}{4})$

$(-2, \frac{11\pi}{4})$

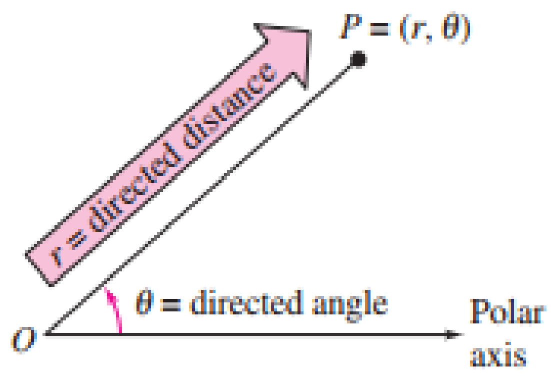



Find 3 more ways to represent:

$$(4, \frac{\pi}{6})$$

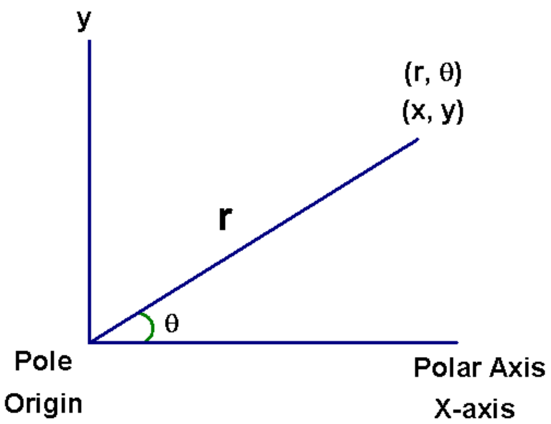
1. $r$ = *directed distance* from $O$ to $P$

2. $\theta$ = *directed angle*, counterclockwise from the polar axis to segment $\overline{OP}$



# Coordinate Conversion

Rectangular to Polar




$$\tan(\theta) = \frac{y}{x}$$

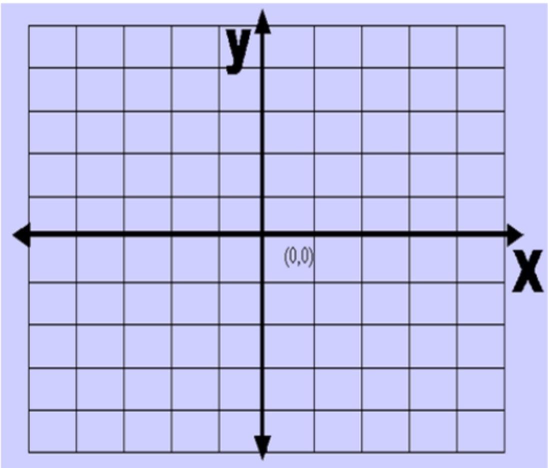
$$r^2 = x^2 + y^2$$

Convert to polar coordinates:

(2, 2)

Plot the point. Then find two sets of polar coordinates for the point for $0 \leq \theta < 2\pi$.

(0, -5)



## Polar to Rectangular

$$x = r\cos(\theta)$$

$$y = r\sin(\theta)$$

## Rectangular to Polar Coordinates

$$\tan(\theta)=\frac{y}{x}$$

$$r^2=x^2+y^2$$

### Coordinate Conversion

The polar coordinates $(r, \theta)$ are related to the rectangular coordinates $(x, y)$ as follows.

| *Polar-to-Rectangular* | *Rectangular-to-Polar* |
|---|---|
| $x = r\cos\theta$ | $\tan\theta = \dfrac{y}{x}$ |
| $y = r\sin\theta$ | $r^2 = x^2 + y^2$ |

## Polar-to-Rectangular Conversion

Convert the point $(2, \pi)$ to rectangular coordinates.

For the point $(r, \theta) = (2, \pi)$, you have the following.

$$x = r\cos\theta = 2\cos\pi = -2$$

$$y = r\sin\theta = 2\sin\pi = 0$$

The rectangular coordinates are $(x, y) = (-2, 0)$. (See Figure 9.65.)

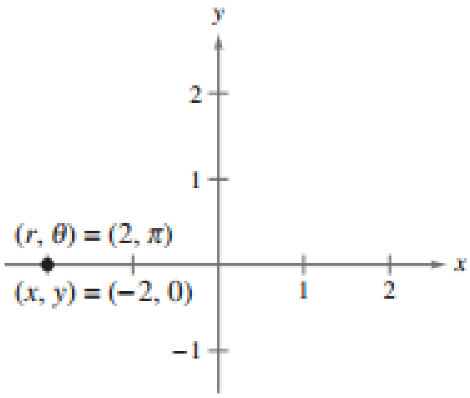



Figure 9.65

# Rectangular-to-Polar Conversion

Convert the point $(-1, 1)$ to polar coordinates.

For the second-quadrant point $(x, y) = (-1, 1)$, you have

$$\tan\theta = \frac{y}{x} = \frac{1}{-1} = -1$$

$$\theta = \frac{3\pi}{4}.$$

Because $\theta$ lies in the same quadrant as $(x, y)$, use positive $r$.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$

So, *one* set of polar coordinates is

$$(r, \theta) = \left(\sqrt{2}, \frac{3\pi}{4}\right)$$

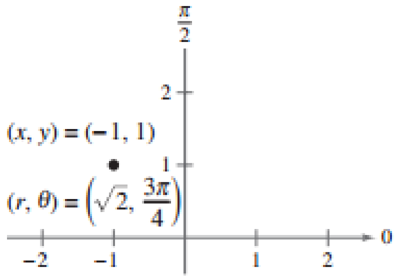


Figure 9.66

Practice Problem:

$x^2 + y^2 = 16$

What does $x^2 + y^2 = ?$

$x^2 + y^2 = r^2$

$r^2 = 16$

$r = 4$