KEY CONCEPT *For Your Notebook*

Definition of Logarithm with Base b

Let b and y be positive numbers with $b \neq 1$. The **logarithm of y with base b** is denoted by $\log_b y$ and is defined as follows:

$$\log_b y = x \quad \text{if and only if} \quad b^x = y$$

The expression $\log_b y$ is read as "log base b of y."

$2^3 = 8$ $\log_2 8 = 3$

$3^2 = 9$ $\log_3 9 = 2$

What is a logarithm?

A logarithm is an EXPONENT

Logarithmic form

$y = \log_a x$

$\log_a x = y$

equivalent

it is said, "log base *a* of *x* is *y*."

Exponential form $a^y = x$

$\therefore \quad \log_a x = y$ and $a^y = x$

Therefore

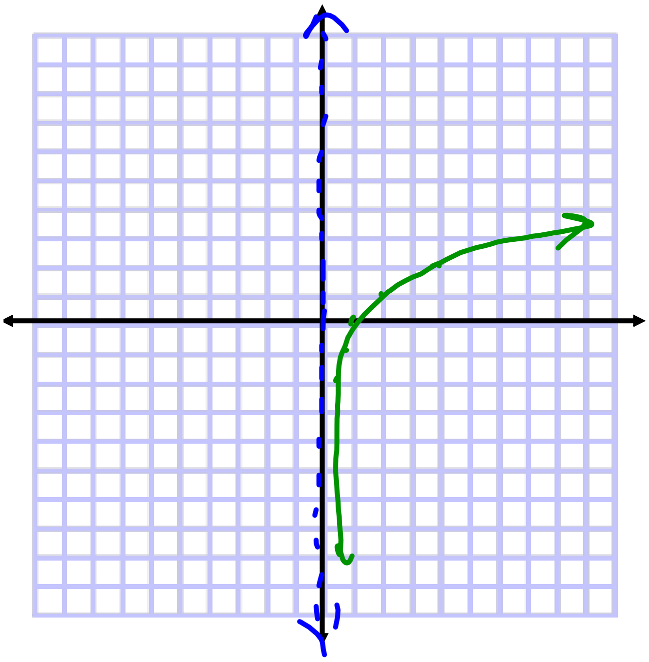
ARE EQUIVALENT

EXAMPLE 1 Rewrite logarithmic equations

	Logarithmic Form	Exponential Form
a.	$\log_2 8 = 3$	$2^3 = 8$
b.	$\log_4 1 = 0$	$4^0 = 1$
c.	$\log_{12} 12 = 1$	$12^1 = 12$
d.	$\log_{1/4} 4 = -1$	$\left(\frac{1}{4}\right)^{-1} = 4$

Logarithmic Functions and Their **Graphs**
Graphs of logarithms:

$y = \log_2 x$



Domain= (0, +∞)
Range= (-∞, +∞)
asymptote x = 0

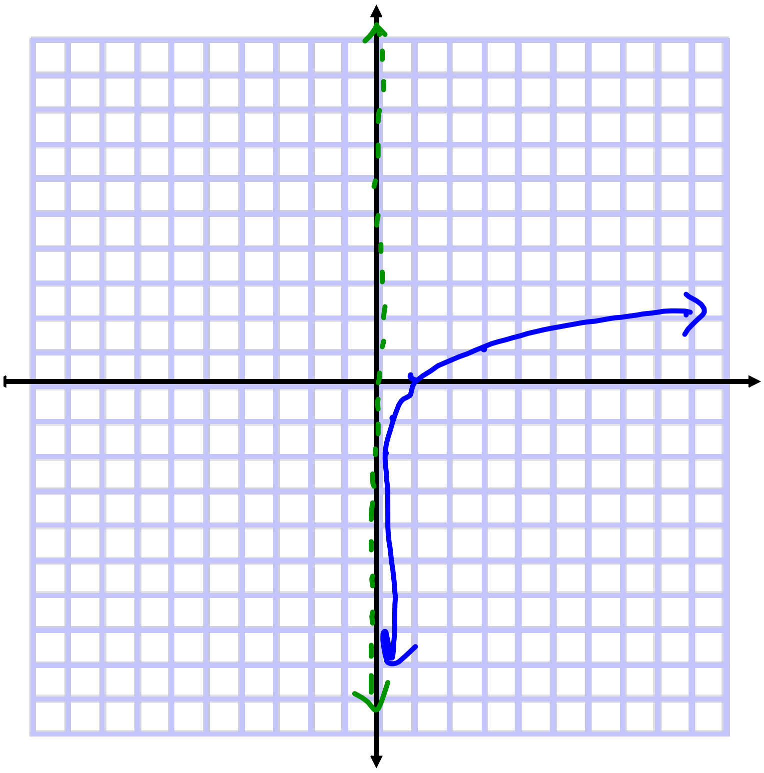
Rewrite as an exponential

$y = \log_2 x$ $2^y = x$

$x=2^y$

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2

$y = \log_3 x$ $3^y = x$



Domain= $(0, +\infty)$

Range= $(-\infty, +\infty)$

Asymptote $x = 0$

What is the inverse?

$y = 3^x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

Graph of $f(x) = \log_a x,\ a > 1$

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Intercept: $(1, 0)$

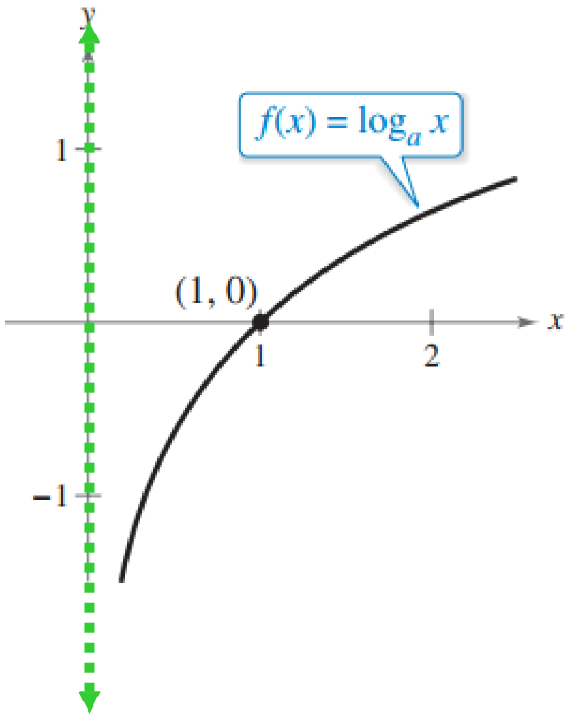
Increasing on $(0, \infty)$

y-axis is a vertical asymptote
($\log_a x \to -\infty$ as $x \to 0^+$)

Continuous

Reflection of graph of $f(x) = a^x$
in the line $y = x$

Asymptote x=0



TRANSLATIONS You can graph a logarithmic function of the form $y = \log_b (x - h) + k$ by translating the graph of the parent function $y = \log_b x$.

EXAMPLE 8 Translate a logarithmic graph

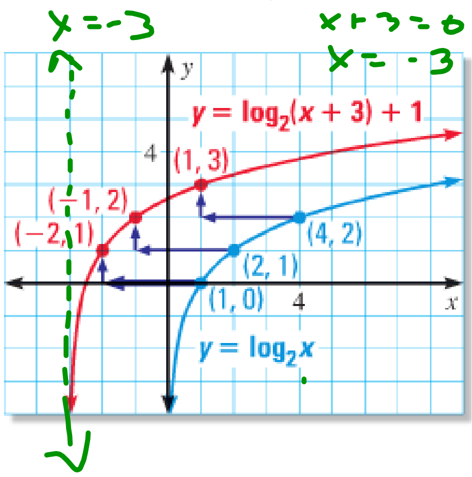
Graph $y = \log_2 (x + 3) + 1$. State the domain and range.

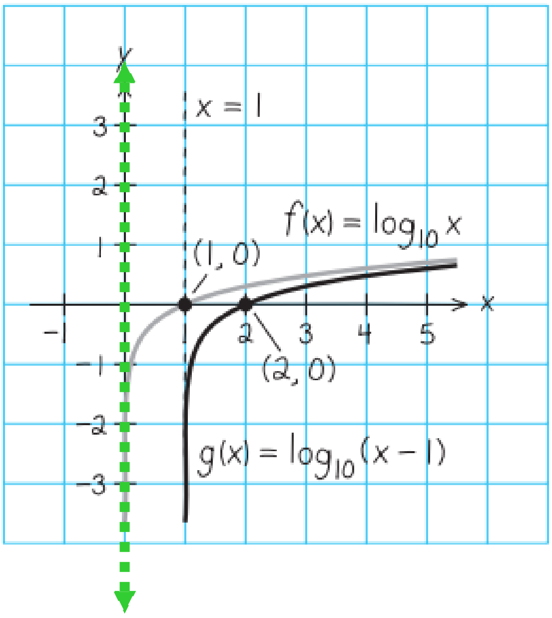
$\log_b (x-h)+k$

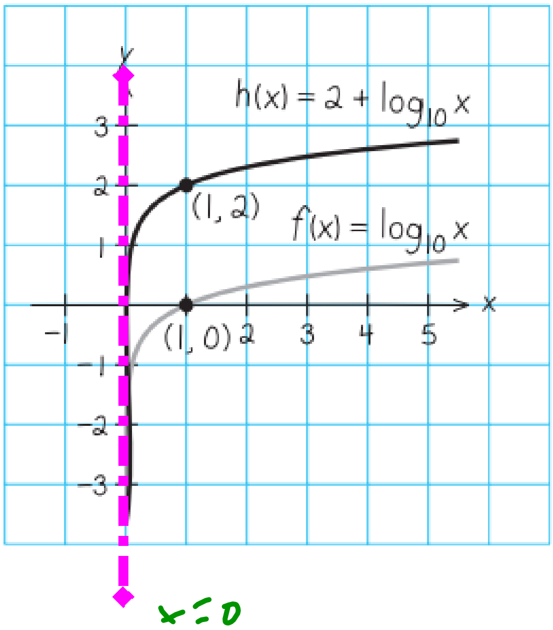
Solution

STEP 1 **Sketch** the graph of the parent function $y = \log_2 x$, which passes through (1, 0), (2, 1), and (4, 2).

STEP 2 **Translate** the parent graph left 3 units and up 1 unit. The translated graph passes through (−2, 1), (−1, 2), and (1, 3). The graph's asymptote is $x = -3$. The domain is $x > -3$, and the range is all real numbers.




y
x = 1
3
2
1
-1
-2
-3
f(x) = log10 x
(1, 0)
(2, 0)
g(x) = log10(x − 1)
x


y
h(x) = 2 + log10 x
(1, 2)
f(x) = log10 x
(1, 0)
x
x = 0

a. $f(x) = 2^x$

b. $g(x) = \log_2 x$

Solution

a. For $f(x) = 2^x$, construct a table of values. By plotting these points and connecting them with a smooth curve, you obtain the graph of f shown in Figure 3.16.

x	-2	-1	0	1	2	3
$f(x) = 2^x$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

b. Because $g(x) = \log_2 x$ is the inverse function of $f(x) = 2^x$, the graph of g is obtained by plotting the points $(f(x), x)$ and connecting them with a smooth curve. The graph of g is a reflection of the graph of f in the line $y = x$, as shown in Figure 3.16.

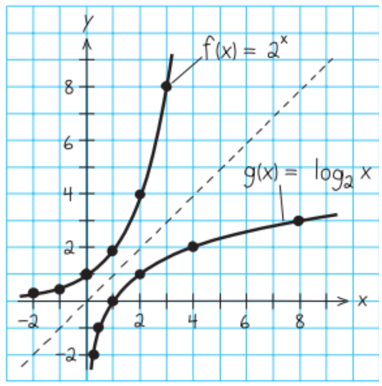
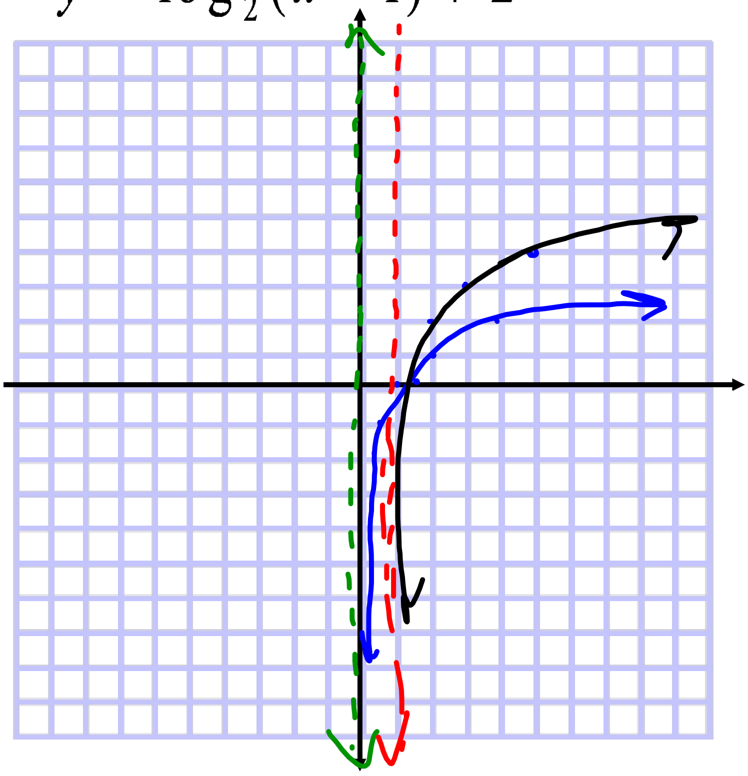


Figure 3.16

$y = \log_2(x - 1) + 2$



Parent Function $y = \log_2 x$

Domain $(1, +\infty)$

Range $(-\infty, +\infty)$

Asymptote $x = 1$

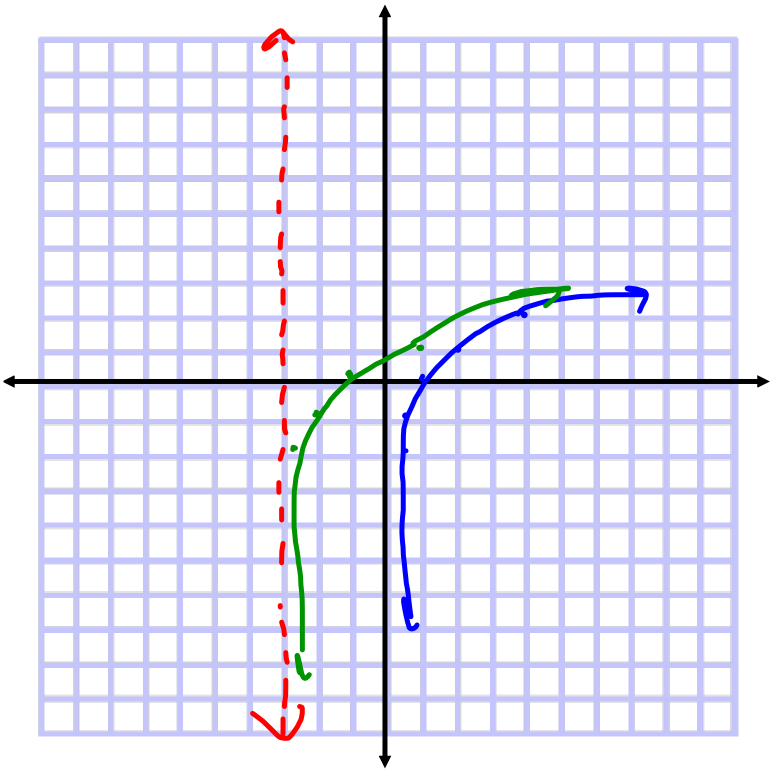
Horizontal Shift Right 1

Vertical Shift Up 2

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2

$2^y = x$

$f(x) = \log_2(x+3) - 1$



Parent Function $y = \log_2 x$

Domain $(-3, +\infty)$

Range $(-\infty, +\infty)$

Asymptote $x = -3$

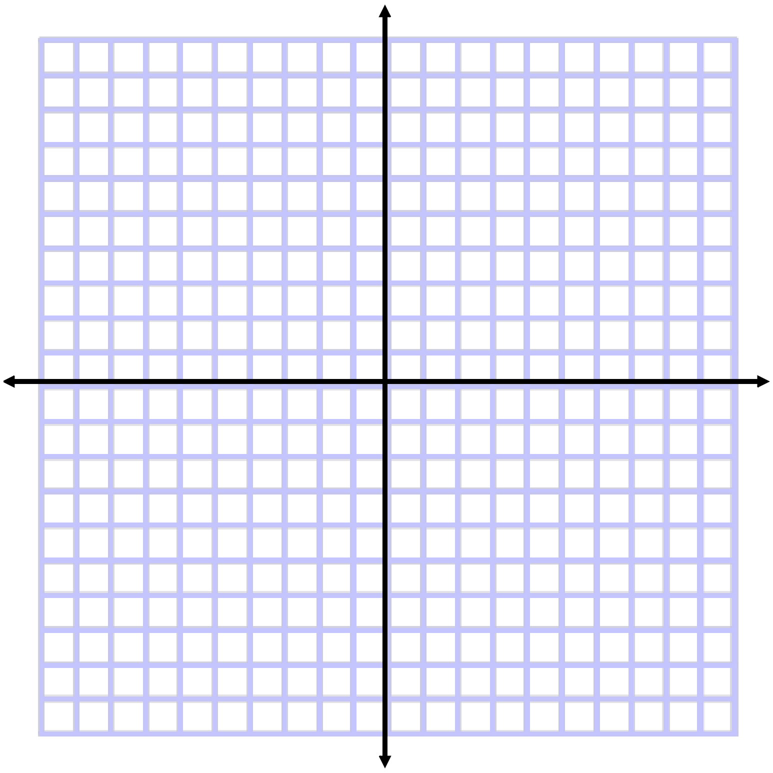
Horizontal Shift Left 3

Vertical Shift Down 1

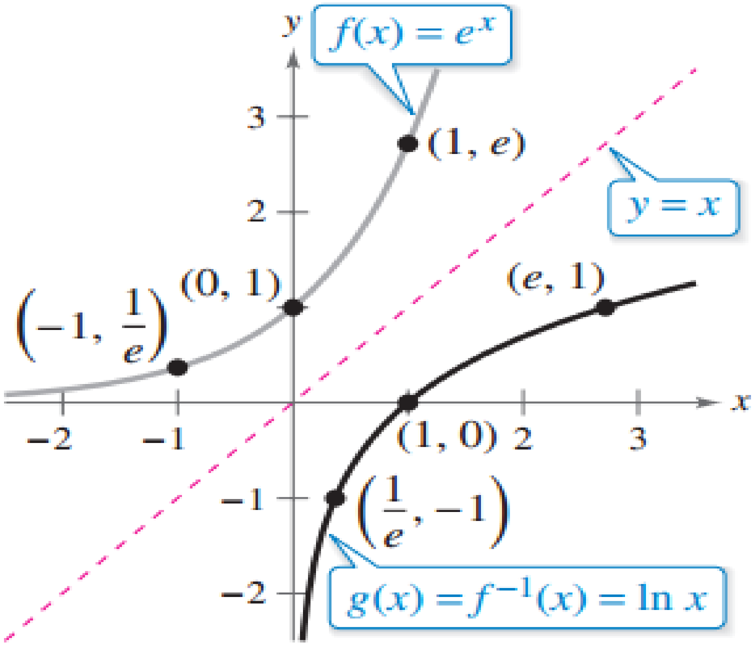
x	y
1/4	-2
1/2	-1
1	0
2	1
4	2

$2^y = x$

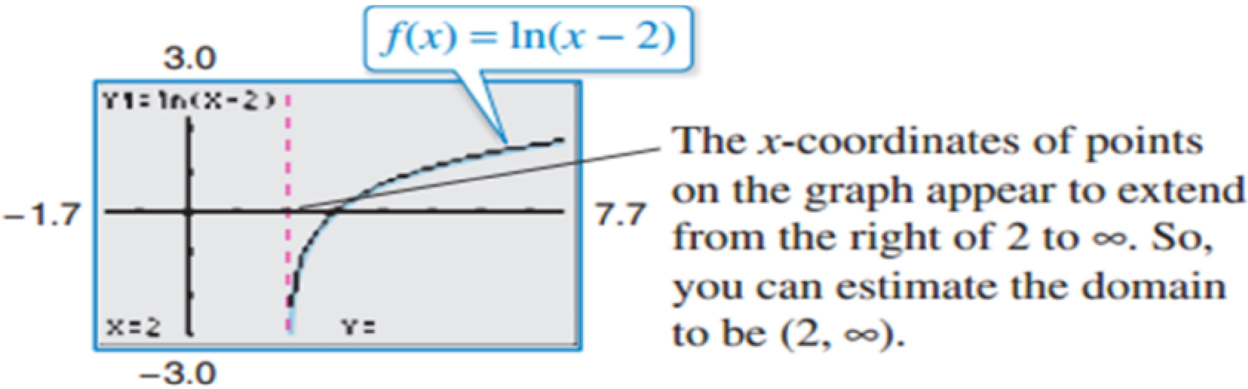
$f(x) = \log(x + 5) - 2$



Parent Function____
Domain__________
Range___________
Asymptote_________
Horizontal Shift ___
Vertical Shift _____



Reflection of graph of $f(x) = e^x$ in the line $y = x$



When graphing with base e, same rules apply

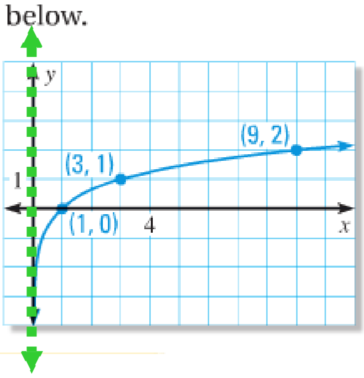
Graph the function.

a. $y = \log_3 x$

Solution

a. Plot several convenient points, such as (1, 0), (3, 1), and (9, 2). The *y*-axis is a vertical asymptote.

From *left* to *right*, draw a curve that starts just to the right of the *y*-axis and moves up through the plotted points, as shown below.



Without Graphing. Find the following

a. $y = \log_4(x-7)+5$

b. $y = \log_6(x+3)-8$

Parent Function $y = \log_4 x$

Domain $(7, +\infty)$

Range $(-\infty, +\infty)$

Asymptote $x = 7$

Horizontal Shift Right 7

Vertical Shift Up 5