Without Graphing Describe each of the following.

$$f(x) = 5\left(\frac{1}{4}\right)^{x-8} + 6$$

$$f(x) = \frac{1}{2}(4)^{x+3} - 5$$

Parent Function $y = 4^x$

Asymptote $y = -5$

Domain $(-\infty, +\infty)$

Range $(-5, +\infty)$

Horizontal Shift Left 3

Vertical Shift Down 5

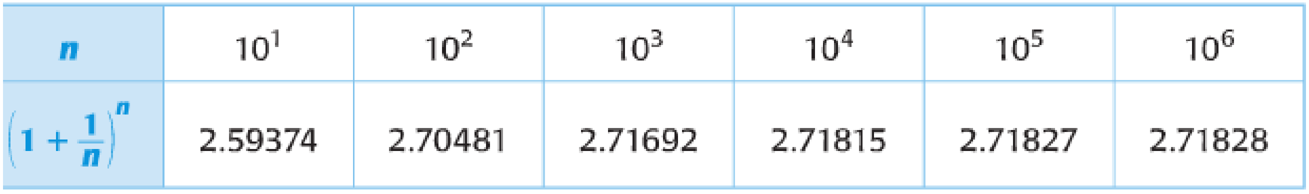
[✓ Growth] or Decay

Vertical Stretch or [✓ Shrink]

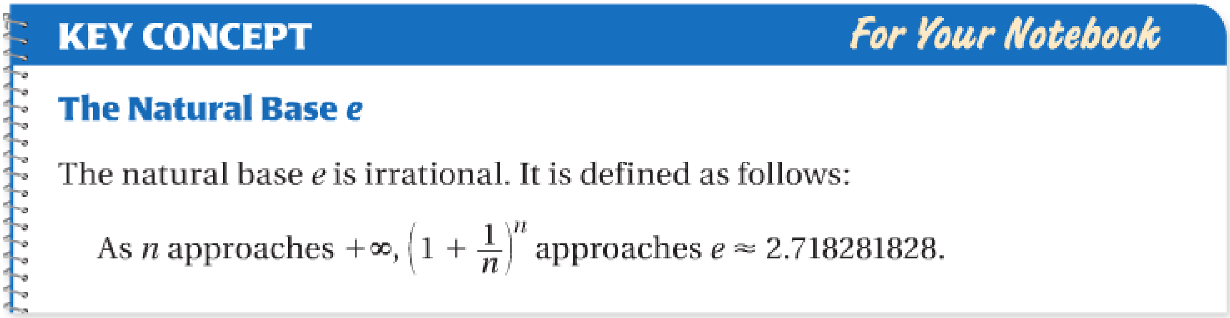
The history of mathematics is marked by the discovery of special numbers such as π and i.

Another special number is denoted by the letter e.

The number is called the natural base e or the Euler number.



n	10^1	10^2	10^3	10^4	10^5	10^6
$\left(1+\frac{1}{n}\right)^n$	2.59374	2.70481	2.71692	2.71815	2.71827	2.71828

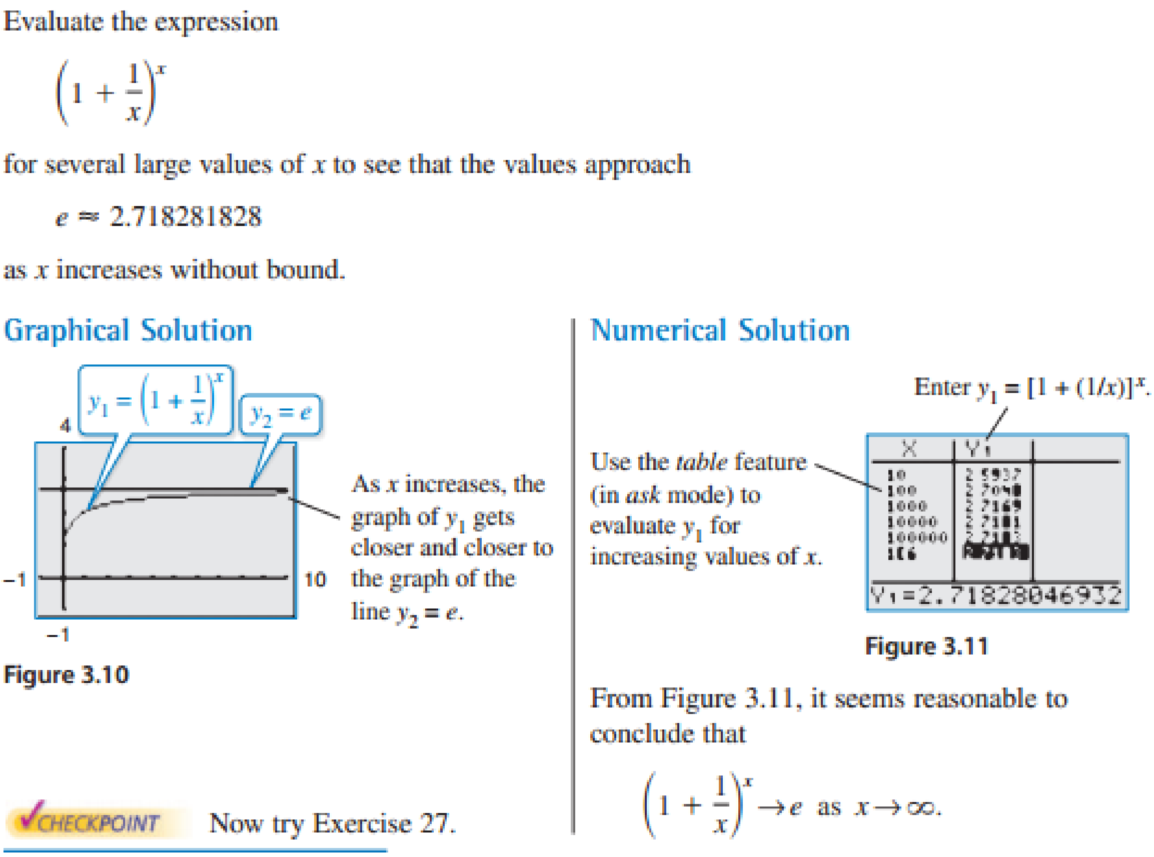


KEY CONCEPT *For Your Notebook*

The Natural Base *e*

The natural base e is irrational. It is defined as follows:

As n approaches $+\infty$, $\left(1+\frac{1}{n}\right)^n$ approaches $e \approx 2.718281828$.



Evaluate the expression

$$\left(1 + \frac{1}{x}\right)^x$$

for several large values of x to see that the values approach

$$e \approx 2.718281828$$

as x increases without bound.

Graphical Solution

As x increases, the graph of y_1 gets closer and closer to the graph of the line $y_2 = e$.

Figure 3.10

CHECKPOINT Now try Exercise 27.

Numerical Solution

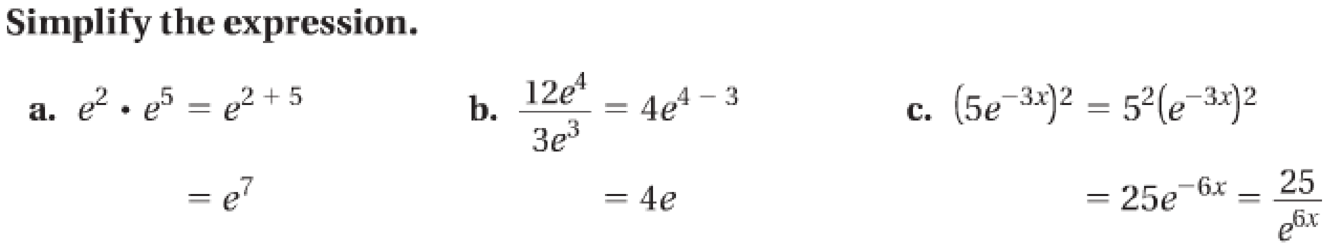
Enter $y_1 = [1 + (1/x)]^x$.

Use the *table* feature (in *ask* mode) to evaluate y_1 for increasing values of x.

Figure 3.11

From Figure 3.11, it seems reasonable to conclude that

$$\left(1 + \frac{1}{x}\right)^x \rightarrow e \text{ as } x \rightarrow \infty.$$



Simplify the expression.

a. $e^2 \cdot e^5 = e^{2+5}$

$= e^7$

b. $\frac{12e^4}{3e^3} = 4e^{4-3}$

$= 4e$

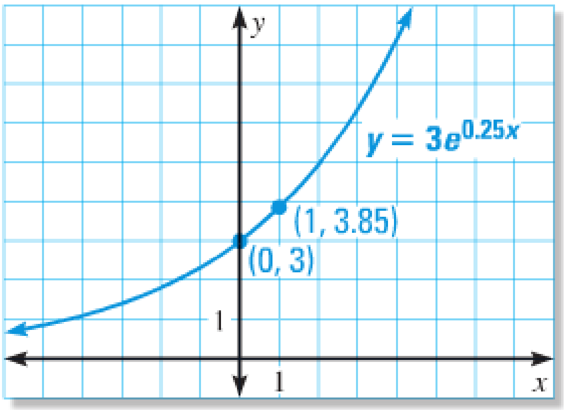
c. $(5e^{-3x})^2 = 5^2(e^{-3x})^2$

$= 25e^{-6x} = \frac{25}{e^{6x}}$

a. $y = 3e^{0.25x}$

Solution

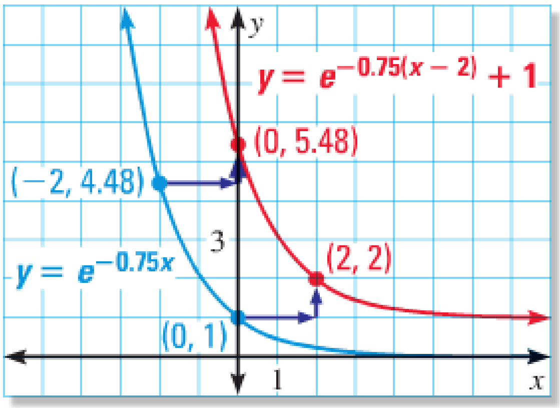
a. Because $a = 3$ is positive and $r = 0.25$ is positive, the function is an exponential growth function. Plot the points (0, 3) and (1, 3.85) and draw the curve.



The domain is all real numbers, and the range is $y > 0$.

b. $y = e^{-0.75(x-2)} + 1$

b. $a = 1$ is positive and $r = -0.75$ is negative, so the function is an exponential decay function. Translate the graph of $y = e^{-0.75x}$ right 2 units and up 1 unit.



The domain is all real numbers, and the range is $y > 1$.

Compound interest: Interest paid on the initial investment called the principal and on previously earned interest.

Simple interest: Interest paid only on the principal.

4 Formulas

you will need to have memorized for Test 6a

Exponential Growth

$$y = a(1+r)^{t}$$

Exponential Decay

$$y = a(1-r)^{t}$$

Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Continuously Compounded Interest

$$A = Pe^{rt}$$

Exponential Growth

$$y = a(1+r)^t$$

Exponential Decay

$$y = a(1-r)^t$$

a represents: principal (initial amount)

r represents: rate as a decimal

t represents: time

y represents: __________

Exponential Growth

$$y = a(1 + r)^t$$

Exponential Decay

$$y = a(1 - r)^t$$

How are these formulas the same and different?

How is the base of each formula affected by the differences?

The value of a snowmobile has been decreasing by 7% each year since it was new. After 3 years, the value is \$3000. Find the original cost of the snowmobile.

$$y = a(1-r)^t$$

$$3000 = a(1-.07)^3$$

$$\frac{3000}{.804} = \frac{a(.804)}{.804}$$

\$3731.34

COMPUTERS In 1996, there were 2573 computer viruses and other computer security incidents. During the next 7 years, the number of incidents increased by about 92% each year.

$y = 2573(1+.92)^7$

Is this growth or decay?

247484.72

247485

How many incidents were there in 2003?

Dare Devil #3 was bought in 1987 for $15.
It's value increases every year at 12%.

$y = 15(1+.12)^{33}$

How much is it worth this year?

$\$631.37$

I want to sell it for $1000.

What year will it be worth $1000?

$1000 = 15(1+.12)^{t}$

SNOWMOBILES A new snowmobile costs $4200. The value of the snowmobile decreases by 10% each year.

Is this growth or decay?

$y = 4200(1-.10)^4$ $y = 2755.62$

How much will the snowmobile be worth after 4 years?

When will the snowmobile be worth 1/3 its original price (hint: graph and find a point when y is 1/3 the original cost)?

Compounded Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

A = amount after time

P = principal (initial amount)

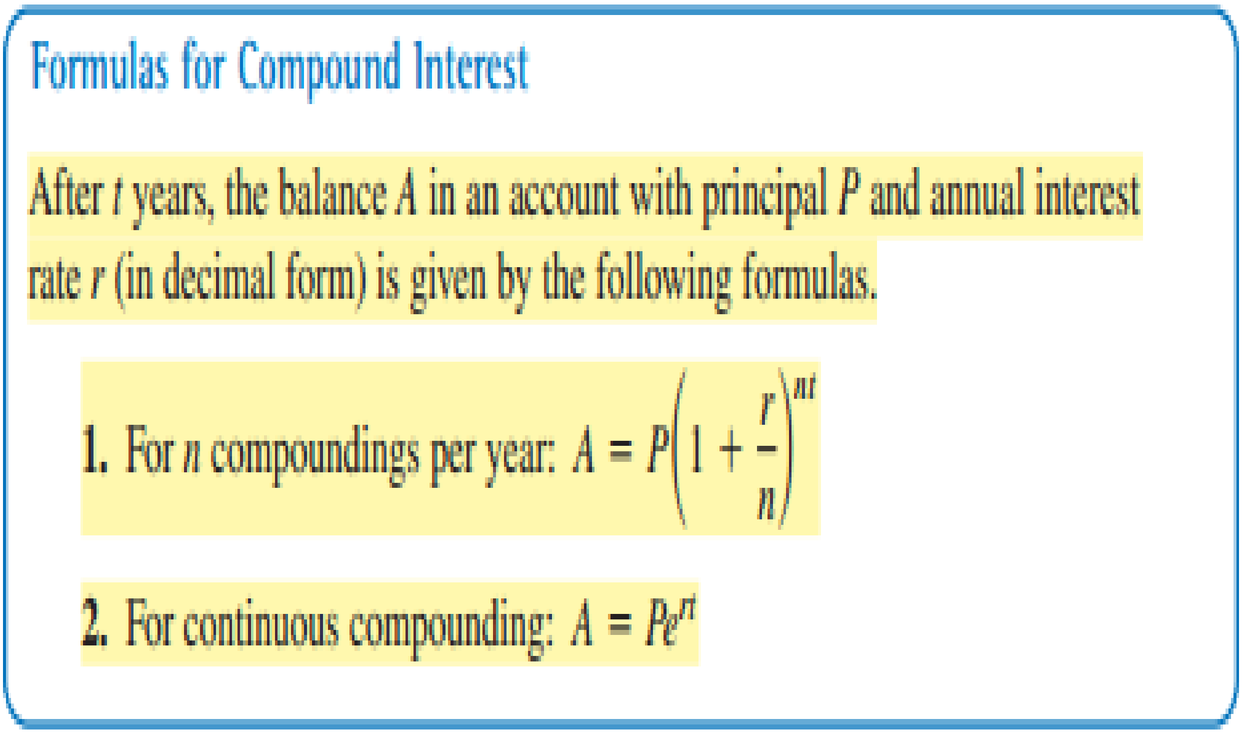
r = interest rate (as a decimal)

n = # times compounded per year

t = time

Interest Compounded Continuously:

$$A = Pe^{rt}$$



Formulas for Compound Interest

After t years, the balance A in an account with principal P and annual interest rate r (in decimal form) is given by the following formulas.

1. For n compoundings per year: $A = P\left(1 + \frac{r}{n}\right)^{nt}$

2. For continuous compounding: $A = Pe^{rt}$

You deposit \$5500 in an account that pays 3.6% annual interest. Find the balance after 2 years if interest is compounded with the given frequency.

Compound Interest

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

a. semiannually

b. monthly

$$A = 5500\left(1+\frac{.036}{12}\right)^{12(2)}$$

$A = \$5906.82$ $\$5909.97$

$\$406.82$

yearly = 1
Quarterly = 4
Monthly = 12
Daily = 365

You deposit $4000 in an account that pays 2.29% annual interest. Find the balance after 5 years if the interest is compounded with the given frequency:

a. Yearly
b. Quarterly
c. Monthly
d. Daily

You put $ 20,000 in an account for 5 years at 6 % interest compounded:

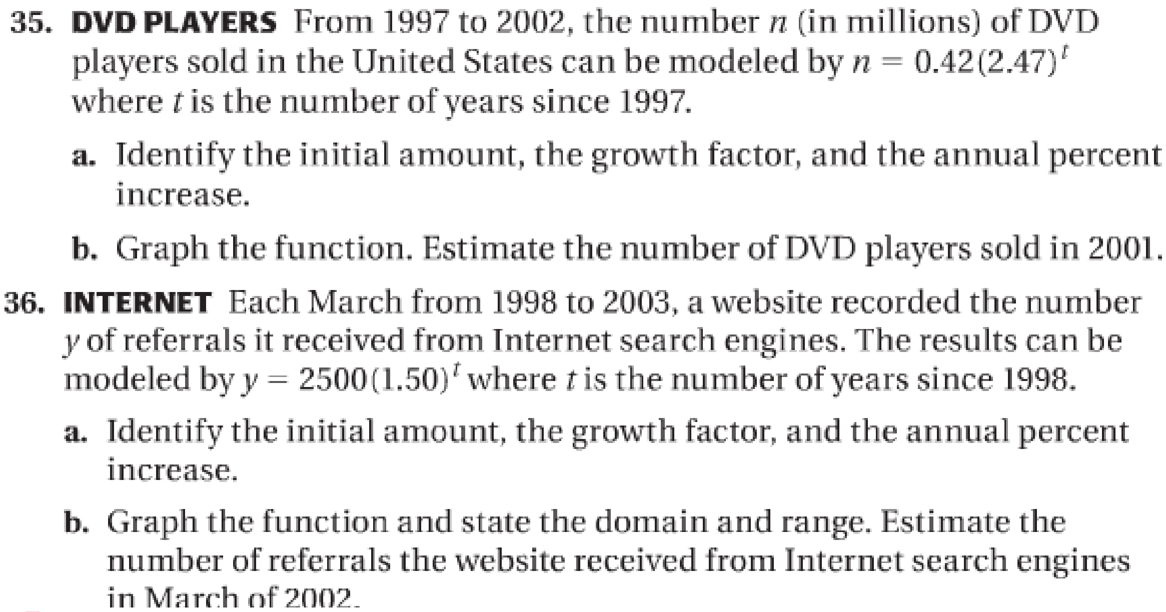
annually	monthly	quarterly	daily	continuously

Which account will pay the most?

A lot more, or only a little more?

WHY??

Class Work: pg 484 (#35 - 38) pg 490 (#31, #33)



35. **DVD PLAYERS** From 1997 to 2002, the number n (in millions) of DVD players sold in the United States can be modeled by $n = 0.42(2.47)^t$ where t is the number of years since 1997.

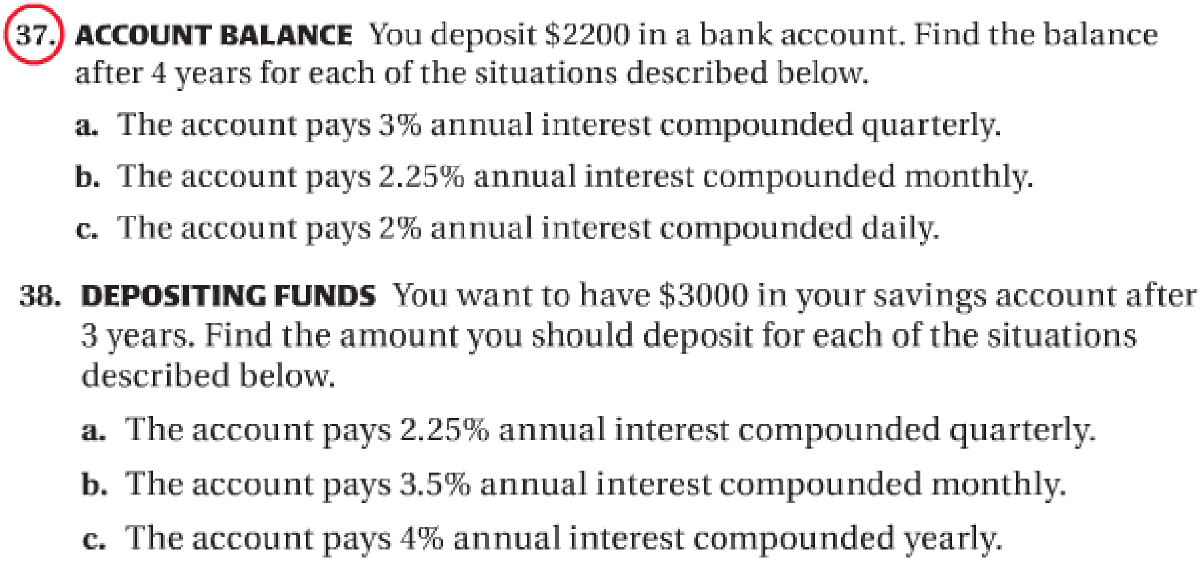
a. Identify the initial amount, the growth factor, and the annual percent increase.

b. Graph the function. Estimate the number of DVD players sold in 2001.

36. **INTERNET** Each March from 1998 to 2003, a website recorded the number y of referrals it received from Internet search engines. The results can be modeled by $y = 2500(1.50)^t$ where t is the number of years since 1998.

a. Identify the initial amount, the growth factor, and the annual percent increase.

b. Graph the function and state the domain and range. Estimate the number of referrals the website received from Internet search engines in March of 2002.



37. **ACCOUNT BALANCE** You deposit $2200 in a bank account. Find the balance after 4 years for each of the situations described below.

a. The account pays 3% annual interest compounded quarterly.

b. The account pays 2.25% annual interest compounded monthly.

c. The account pays 2% annual interest compounded daily.

38. **DEPOSITING FUNDS** You want to have $3000 in your savings account after 3 years. Find the amount you should deposit for each of the situations described below.

a. The account pays 2.25% annual interest compounded quarterly.

b. The account pays 3.5% annual interest compounded monthly.

c. The account pays 4% annual interest compounded yearly.

31. **BIKE COSTS** You buy a new mountain bike for $200. The value of the bike decreases by 25% each year.

    a. Write a model giving the mountain bike's value y (in dollars) after t years. Use the model to estimate the value of the bike after 3 years.

    b. Graph the model.

    c. Estimate when the value of the bike will be $100.

33. ★ **SHORT RESPONSE** The value of a car can be modeled by the equation $y = 24{,}000(0.845)^t$ where t is the number of years since the car was purchased.

    a. Graph the model. Estimate when the value of the car will be $10,000.

    b. Use the model to predict the value of the car after 50 years. Is this a reasonable value? *Explain.*