Warm up:

Use the limit definition to find the derivative:

$f(x) = 3x^2 + 2x^1$

$6x + 2$

Can you determine a shortcut for finding the derivative of this function?

$$\frac{3(x+h)^2 + 2(x+h) - (3x^2 + 2x)}{h}$$

$3(x^2 + 2xh + h^2)$ $+2x + 2h$

$$\frac{3x^2 + 6xh + 3h^2 - 3x^2 + 2x}{h}$$

$$\frac{6xh + 3h^2 + 2h}{h}$$

$$\frac{h(6x + 3h + 2)}{h}$$

$m = 6x + 2$

$\frac{d}{dx}(x)_{x=x}$

x^1

$y=1$

Discovering Derivative Properties

For each of the following problems, a function $f(x)$ is given. You are to try to discover the function $f'(x)$ by finding a match for the calculator generated function **nDeriv(Y1, X, X)**.

1. Put the given function $f(x)$ into **Y1** of your graphing calculator. (You may want to turn it off by deactivating it.)
2. Let **Y2** = **nDeriv(Y1, X, X)**.
3. Guess the function that you see in **Y2** and check you guess by putting it into **Y3**.
4. If it matches, record your answer; if it doesn't, try again!
5. Don't forget, we are looking for patterns and generalizations that we can write as a property.

$4x^2$
$8x$

Property (The Power Rule):

If $f(x) = x^n$, then $f'(x) =$ nx^{n-1}

Property:

If k is a number and $f(x) = k * g(x)$,

then $f'(x) =$ Knx^{n-1}

Property:

If k is a number and $f(x) = k$, then $f'(x) =$ 0

$3x^0$

Property:

If $f(x) = g(x) + k(x)$, then $f'(x) =$ $g'(x) + k'(x)$

$6x^3 + 4x^2 + 6$
$18x^2 + 8x$

Question: Does the Power Rule hold for other numbers besides whole numbers?

Negative whole numbers? yes

Rational whole numbers? yes

$\frac{1}{x} = x^{-1}$
$-x^{-2} = \frac{-1}{x^2}$

$\sqrt{x} = x^{1/2 - 1} = \frac{1}{2}x^{-1/2}$

$\sqrt[3]{x^2} = x^{2/3}$

Power Rule to find a derivative

$$\frac{d}{dx}[x^n] = nx^{n-1}$$

you say "n times *x* to the n -1"

Do you get the same answer?

$$f(x) = 3x^2 + 2x$$

What is a derivative of a constant?

4

Why?

Derivative of a Constant Function

Using the power rule formula, we find that the derivative of a function that is a constant would be zero.

For any constant c,

The derivative of $f(x) = c$ is $f'(x) = 0$

which can also be written as

$$\frac{d}{dx}(c) = 0$$

Example:

Differentiate the following:

a) $f(3)$

b) $f(157)$

$$\frac{d}{dx}[c] = 0$$

The derivative of a constant = 0 (slope)

Examples: (sketch a graph)

$y = 7$

$f(x) = -3$

$g(x) = 0$

What is a derivative of a variable with a power of 1?

x^1

Why?

Derivative of the function $f(x) = x$

Using the power rule formula, we find that the derivative of the function $f(x) = x$ would be one.

The derivative of $f(x) = x$ is $f'(x) = 1$

which can also be written as

$$\frac{d}{dx}(x) = 1$$

Example:

Differentiate $f(x) = x$

Solution:

$f'(x) = f'(x^1) = 1x^0 = 1$

$$\frac{d}{dx}[x] = 1$$

The derivative of x = 1 (slope)

Example:

$y = x$

$f(x) = 5x$

$h(x) = -2x$

The Constant Multiple Rule

The constant multiple rule says that the derivative of a constant value times a function is the constant times the derivative of the function.

If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Example:

Differentiate the following:

a) $y = 2x^4$

b) $y = -x$

Solution:

$$\text{a) } \frac{d}{dx}\left(2x^4\right) = 2\frac{d}{dx}x^4 = 2(4x^3) = 8x^3$$

$$\text{b) } \frac{d}{dx}(-x) = \frac{d}{dx}\left[(-1)x\right] = -1\frac{d}{dx}x = -1(1) = -1$$

The Sum Rule

The Sum Rule tells us that the derivative of a sum of functions is the sum of the derivatives.

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

The Sum Rule can be extended to the sum of any number of functions.
For example $(f + g + h)' = f' + g' + h'$

Example:

Differentiate $5x^2 + 4x + 7$

Solution:

$$\frac{d}{dx}\left[5x^2+4x+7\right]=\frac{d}{dx}\left(5x^2\right)+\frac{d}{dx}(4x)+\frac{d}{dx}(7)=10x+4+0=10x+4$$

The Difference Rule

The Difference Rule tells us that the derivative of a difference of functions is the difference of the derivatives.

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

Example:

Differentiate $x^8 - 5x^2 + 6x$

Solution:

$$\frac{d}{dx}\left[x^8-5x^2+6x\right]=\frac{d}{dx}\left(x^8\right)-\frac{d}{dx}\left(5x^2\right)+\frac{d}{dx}(6x)=8x^7-10x+6$$

Examples

a) $f(x) = x^3$

$3x^2$

b) $y = x^{100}$

$100x^{99}$

c) $g(x) = \sqrt{x}$

$x^{1/2}$

$1/2 x^{-1/2}$

d) $f(x) = \dfrac{1}{x^4}$

x^{-4}

$-4x^{-5}$

$$\frac{d}{dx}[c\, f(x)] = c\, f'(x)$$

you can pull out scalars

Original Function

Simplfy

$$f(x) = \frac{5x^2}{2}$$

$$y = \frac{7}{(2x)^4}$$

$$y = 4\sqrt{x}$$

stop practice 3-17 off (11-17 do a term at a time)

you can add and subtract derivatives

$\frac{d}{dx}[f(x)+g(x)]=f'(x)+g'(x)$	$\frac{d}{dx}[f(x)-g(x)]=f'(x)-g'(x)$

it just means to power rule to each term

a) $f(x) = 2x^3 - 3x + 8$

b. $g(x) = \frac{x^5}{2} - 4x^3 + 2x$

Let's review the first two rules, now using the power rule.

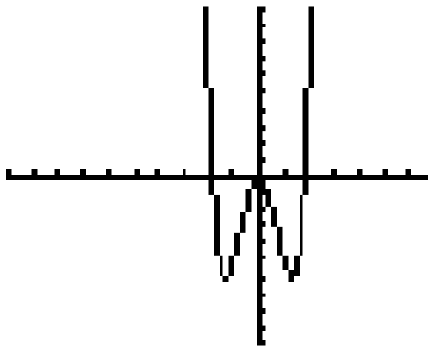
a) $y = 2x$

b) $y = 3$

Example 1

Find the slope of the graph

$f(x)= 2x^4 -7x^2$ when $x = -1$, $x = 0$, $x = 1$



Example 2

Find the equation of the tangent line to

f(x)= 3x^2 when x = -1

Position Function

$$s(t) = \frac{1}{2}gt^2 + v_o t + s_o$$

v_o = initial velocity

s_o = initial height

t = time

g = gravity

- 32 ft/sec^2

or

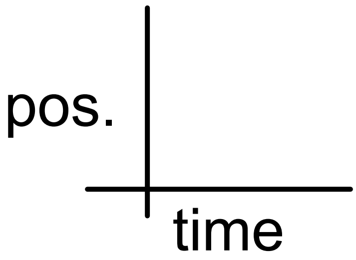
- 9.8 m/sec^2

Average Velocity vs. Instaneous Velocity

Average Velocity

Algebra slope
(no calculus needed)

$$\frac{\Delta s}{\Delta t} = \frac{\text{position}}{\text{time}}$$

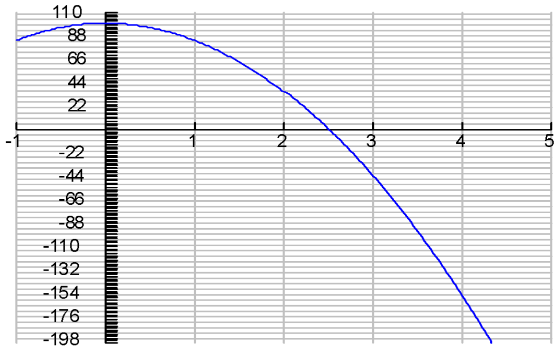


Instaneous Velocity

s '(t)

take the derivative

$s(t) = -16t^2 + 100$



a) Find the average velocity from [2, 4]

b) Find the instantaneous velocity at t = 2

A penny falling is given by the function

$$s(t) = -16t^2 + 32t + 48$$

a) When does it hit the ground?

Graph it on your graphing calculator.

b) What is the velocity when it hits the ground?

In your own words, describe the difference between ***average velocity*** and ***instantaneous velocity***.