Warm Up

Differentiate each function with respect to *x*.

1) $y = -5\sqrt[4]{x} - 5x^{-1} - 5x^{-2}$

2) $y = -2\sqrt[3]{x^2} + 2x^{\frac{1}{2}} + 3x^{-5}$

3) $f(x) = -x^2 + 3x^{-2} - 4x^{-5}$

4) $y = 2x^2 + 5x^{\frac{2}{5}} - 3x^{-5}$

5) $y = -5x^{\frac{4}{5}} - \sqrt[3]{x^2} + 2\sqrt[4]{x}$

6) $y = x^5 + 2x^2 + \sqrt[3]{x}$

Product Rule

$$\frac{d}{dx}[f(x)g(x)] = [f(x)g'(x) + g(x)f'(x)]$$

"first times derivative of the second plus second times the derivative of the first"

1d2+2d1

Distribute and then use the power rule

$$f(x) = (3x)(x^2 + 1)$$

$3x^3 + 3x$

$f'(x) = 9x^2 + 3$

Now just do the power rule

$$f(x) = (3x)(x^2 + 1)$$

$(3x)(2x) + (x^2+1)(3)$

$6x^2 + 3x^2 + 3$

$9x^2 + 3$

What do you notice?

$$\frac{d}{dx}[f(x)g(x)] = [f(x)g'(x) + g(x)f'(x)]$$

Sometimes it is easier to just use the power rule, and in the future you may have no choice.

$$f(x) = (x^3 - 3x)(2x^2 + 3x + 5)$$

$$f'(x) = (x^3 - 3x)(4x+3) + (2x^2 + 3x + 5)(3x^2 - 3)$$

$$4x^4 + 3x^3 - 12x^2 - 9x + 6x^4 - 6x^2 + 9x^3 - 9x + 15x^2 - 15$$

$$f'(x) = 10x^4 + 12x^3 - 3x^2 - 18x - 15$$

Differentiate each function with respect to *x*.

1) $y = (-4x^3 + 1) \cdot 3x^5$

2) $y = (5x^5 + 5) \cdot 3x^4$

3) $y = (-5x^2 + 3) \cdot -3x^3$

4) $y = -x^5(3x^3 + 5)$

$$\frac{dy}{dx} = -x^5 \cdot 9x^2 + (3x^3 + 5) \cdot -5x^4$$
$$= -24x^7 - 25x^4$$

The product rule is a formal rule for differentiating problems where one function is multiplied by another. The rule follows from the limit definition of derivative and is given by

$$D\{f(x)g(x)\} = f(x)g'(x) + f'(x)g(x)$$

Remember the rule in the following way. Each time, differentiate a different function in the product and add the two terms together.

Function	Derivative
$\sin(x)$	$\cos(x)$
$\cos(x)$	$-\sin(x)$
$\tan(x)$	$\sec^2(x)$
$\cot(x)$	$-\csc^2(x)$
$\sec(x)$	$\sec(x)\tan(x)$
$\csc(x)$	$-\csc(x)\cot(x)$

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \left(\frac{\sin(x)}{\cos(x)}\right)' = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = 1 + \tan^2(x) = \sec^2(x)$$

$$\frac{d}{dx}\cot(x) = \left(\frac{\cos(x)}{\sin(x)}\right)' = \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = -(1 + \cot^2(x)) = -\csc^2(x)$$

$$\frac{d}{dx}\sec(x) = \left(\frac{1}{\cos(x)}\right)' = \frac{\sin(x)}{\cos^2(x)} = \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{\cos(x)} = \sec(x)\tan(x)$$

$$\frac{d}{dx}\csc(x) = \left(\frac{1}{\sin(x)}\right)' = -\frac{\cos(x)}{\sin^2(x)} = -\frac{1}{\sin(x)} \cdot \frac{\cos(x)}{\sin(x)} = -\csc(x)\cot(x)$$