Product and Quotient Rules

Objective: You will be able to:

- use the Product Rule
- use the Quotient Rule

Warm Up

Differentiate each function with respect to x.

1) $y = -5\sqrt[4]{x} - 5x^{-1} - 5x^{-2}$

2) $y = -2\sqrt[3]{x^2} + 2x^{\frac{1}{2}} + 3x^{-5}$

3) $f(x) = -x^2 + 3x^{-2} - 4x^{-5}$

4) $y = 2x^2 + 5x^{\frac{2}{5}} - 3x^{-5}$

5) $y = -5x^{\frac{4}{5}} - \sqrt[3]{x^2} + 2\sqrt[4]{x}$

6) $y = x^5 + 2x^2 + \sqrt[3]{x}$

Differentiate each function with respect to *x*.

1) $y = (x^5 + 3) \cdot 2x^2$

$$\frac{dy}{dx} = (x^5 + 3) \cdot 4x + 2x^2 \cdot 5x^4$$
$$= 14x^6 + 12x$$

2) $y = (-2x^4 + 2)(4x^4 - 2x^2 + 3)$

$$\frac{dy}{dx} = (-2x^4 + 2)(16x^3 - 4x) + (4x^4 - 2x^2 + 3) \cdot -8x^3$$
$$= -64x^7 + 24x^5 + 8x^3 - 8x$$

Stand and Deliver

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

"lo D hi, minus hi D lo, over lo lo"

Examples

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

1. $g(x) = \dfrac{3x - 1}{x^2 + 3}$

$(x^2+3)(x^2+3)$

x^4+6x^2+9

$$\frac{(x^2+3)(3) - (3x-1)(2x)}{x^4+6x^2+9}$$

$$\frac{3x^2+9 - (6x^2 - 2x)}{x^4+6x^2+9}$$

$$\frac{-3x^2+2x+9}{x^4+6x^2+9}$$

Examples

2. $g(x) = \dfrac{\sqrt[3]{x}}{x^3+1}$ $= \dfrac{x^{1/3}}{x^3+1}$

$\frac{1}{3} - \frac{\cancel{3}\,9}{3}$

$3 + \frac{-2}{3}$

$$\frac{(x^3+1)(1/3x^{-2/3}) - (x^{1/3})(3x^2)}{x^6+2x^3+1}$$

$\frac{1}{3} + 2$

$$\frac{1/3x^{7/3} + 1/3x^{-2/3} - 3x^{7/3}}{x^6+2x^3+1} = \frac{-8/3x^{7/3} + 1/3x^{-\frac{2}{3}}}{x^6+2x^3+1}$$

trick question
don't need quotient rule

2. $y = \dfrac{x^2 + 3x}{6}$

$\frac{1}{6}x^2 + \frac{1}{2}x$

Differentiate each function with respect to *x*.

1) $y = \dfrac{x^3}{2x^4 - 4}$

$$\frac{dy}{dx} = \frac{(2x^4 - 4)\cdot 3x^2 - x^3 \cdot 8x^3}{(2x^4 - 4)^2}$$

$$= \frac{-x^6 - 6x^2}{2x^8 - 8x^4 + 8}$$

$$dy/dx = \frac{6x^6 - 12x^2 - 8x^6}{4x^8 - 16x^4 + 16} = \frac{-2x^6 - 12x^2}{4x^8 - 16x^4 + 16}$$

$$\frac{\cancel{2}(-x^6 - 6x^2)}{\cancel{2}(2x^8 - 8x^4 + 8)}$$

2) $y = \dfrac{x^4}{2x^5 + 3}$

$$\frac{dy}{dx} = \frac{(2x^5 + 3)\cdot 4x^3 - x^4 \cdot 10x^4}{(2x^5 + 3)^2}$$

$$= \frac{-2x^8 + 12x^3}{4x^{10} + 12x^5 + 9}$$

3) $y = \dfrac{2x^4}{4x^5 - 5}$

$$\frac{dy}{dx} = \frac{(4x^5 - 5)\cdot 8x^3 - 2x^4 \cdot 20x^4}{(4x^5 - 5)^2}$$

$$= \frac{-8x^8 - 40x^3}{16x^{10} - 40x^5 + 25}$$

4) $y = \dfrac{3x^4}{2x^5 + 2}$

$$\frac{dy}{dx} = \frac{(2x^5 + 2)\cdot 12x^3 - 3x^4 \cdot 10x^4}{(2x^5 + 2)^2}$$

$$= \frac{-3x^8 + 12x^3}{2x^{10} + 4x^5 + 2}$$

2. Use the table below to find the value of $\frac{d}{dx}(fg)$ at x = 3.

(A) 5/2 (B) -3/2 (C) -13 (D) 12 (E) 21/2

$-7 + -6$

$f(x)g'(x) + g(x)f'(x)$

$7 \cdot -1 + -4 \cdot 3/2$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	2	5	$\frac{1}{2}$
3	7	- 4	$\frac{3}{2}$	- 1