

## Product and Quotient Rules

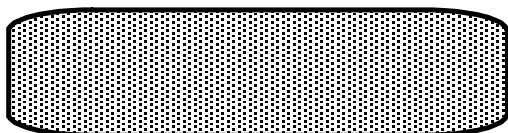
**Objective: You will be able to:**

- use the Product Rule
- use the Quotient Rule

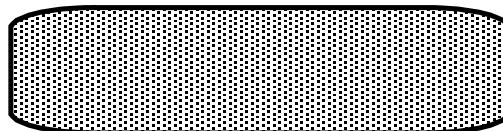
## Warm Up

Differentiate each function with respect to  $x$ .

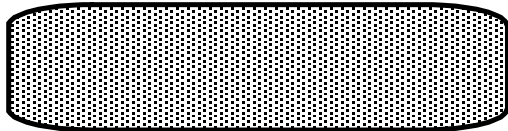
1)  $y = -5\sqrt[4]{x} - 5x^{-1} - 5x^{-2}$



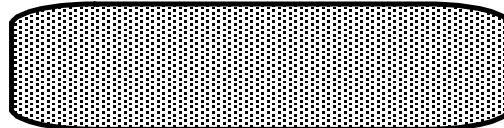
2)  $y = -2\sqrt[3]{x^2} + 2x^{\frac{1}{2}} + 3x^{-5}$



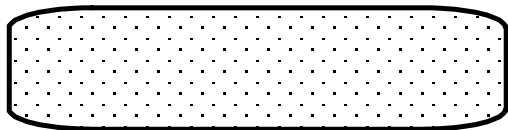
3)  $f(x) = -x^2 + 3x^{-2} - 4x^{-5}$



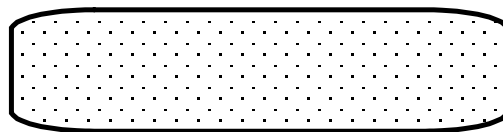
4)  $y = 2x^2 + 5x^{\frac{2}{5}} - 3x^{-5}$



5)  $y = -5x^{\frac{4}{5}} - \sqrt[3]{x^2} + 2\sqrt[4]{x}$



6)  $y = x^5 + 2x^2 + \sqrt[3]{x}$



**Differentiate each function with respect to  $x$ .**

1)  $y = (x^5 + 3) \cdot 2x^2$

$$\begin{aligned}\frac{dy}{dx} &= (x^5 + 3) \cdot 4x + 2x^2 \cdot 5x^4 \\ &= 14x^6 + 12x\end{aligned}$$

2)  $y = (-2x^4 + 2)(4x^4 - 2x^2 + 3)$

$$\begin{aligned}\frac{dy}{dx} &= (-2x^4 + 2)(16x^3 - 4x) + (4x^4 - 2x^2 + 3) \cdot -8x^3 \\ &= -64x^7 + 24x^5 + 8x^3 - 8x\end{aligned}$$

Stand and DeliverQuotient Rule

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

"lo D hi, minus hi D lo, over lo lo"

## Examples

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

$$1. \quad g(x) = \frac{3x - 1}{x^2 + 3}$$

$$(x^2 + 3)(x^2 + 3) \\ x^4 + 6x^2 + 9$$

$$\frac{(x^2 + 3)(3) - (3x - 1)(2x)}{x^4 + 6x^2 + 9}$$

$$\frac{3x^2 + 9 - (6x^2 - 2x)}{x^4 + 6x^2 + 9}$$

$$\frac{-3x^2 + 2x + 9}{x^4 + 6x^2 + 9}$$

### Examples

2.  $g(x) = \frac{\sqrt[3]{x}}{x^3 + 1} = \frac{x^{1/3}}{x^3 + 1}$   $\frac{1}{3} - \frac{3 \cdot 9}{3}$

$3 + \frac{-2}{3}$

$$\frac{(x^3 + 1)(\frac{1}{3}x^{-2/3}) - (x^{1/3})(3x^2)}{x^6 + 2x^3 + 1} = \frac{\frac{1}{3}x^{1/3} - 3x^{5/3}}{x^6 + 2x^3 + 1}$$

$$\frac{\frac{1}{3}x^{1/3} + \frac{1}{3}x^{-2/3} - 3x^{1/3}}{x^6 + 2x^3 + 1} = \frac{-\frac{8}{3}x^{1/3} + \frac{1}{3}x^{-2/3}}{x^6 + 2x^3 + 1}$$

$$2. \quad y = \frac{x^2 + 3x}{6}$$
$$\frac{1}{6}x^2 + \frac{1}{2}x$$

trick question  
don't need quotient rule

Differentiate each function with respect to  $x$ .

$$1) y = \frac{x^3}{2x^4 - 4}$$

$$\frac{dy}{dx} = \frac{(2x^4 - 4) \cdot 3x^2 - x^3 \cdot 8x^3}{(2x^4 - 4)^2}$$

$$= \frac{-x^6 - 6x^2}{2x^8 - 8x^4 + 8}$$

$$\frac{dy}{dx} = \frac{4x^6 - 12x^2 - 8x^6}{4x^8 - 16x^4 + 16} = \frac{-2x^6 - 12x^2}{4x^8 - 16x^4 + 16}$$

$$2) y = \frac{x^4}{2x^5 + 3}$$

$$\frac{dy}{dx} = \frac{(2x^5 + 3) \cdot 4x^3 - x^4 \cdot 10x^4}{(2x^5 + 3)^2}$$

$$= \frac{-2x^8 + 12x^3}{4x^{10} + 12x^5 + 9}$$

$$\frac{2(-x^6 - 6x^2)}{4(2x^8 - 8x^4 + 8)}$$

$$3) y = \frac{2x^4}{4x^5 - 5}$$

$$\frac{dy}{dx} = \frac{(4x^5 - 5) \cdot 8x^3 - 2x^4 \cdot 20x^4}{(4x^5 - 5)^2}$$

$$= \frac{-8x^8 - 40x^3}{16x^{10} - 40x^5 + 25}$$

$$4) y = \frac{3x^4}{2x^5 + 2}$$

$$\frac{dy}{dx} = \frac{(2x^5 + 2) \cdot 12x^3 - 3x^4 \cdot 10x^4}{(2x^5 + 2)^2}$$

$$= \frac{-3x^8 + 12x^3}{2x^{10} + 4x^5 + 2}$$



2. Use the table below to find the value of  $\frac{d}{dx}(fg)$  at  $x = 3$ .

- (A)  $5/2$  (B)  $-3/2$  (C)  $-13$  (D)  $12$  (E)  $21/2$

$$f(x)g'(x) + g(x)f'(x)$$

$$7 \cdot -1 + -4 \cdot \frac{3}{2}$$

$$-7 + -6$$

$$-13$$

$x$	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	4	2	5	$\frac{1}{2}$
3	7	-4	$\frac{3}{2}$	-1

